On the variation of the energy scale 4

Clusters of galaxies

by
Jo. Ke.

Fri 27th Nov 2015

Summary

The hypothesis has been put forward that variations in the energy scale from location to location are the cause of the flat rotation curves observed in most spiral galaxies. The question naturally arises as to whether the same hypothesis can explain the observed velocities of galaxies in clusters of galaxies. A simple spherical model demonstrates that variations in the energy scale can account for the velocities of galaxies in clusters of galaxies. There is no need for changes to Newtonian gravity nor for the existence of dark matter.

1 Introduction

1.1 The paper "On the variation of the energy scale: an alternative to dark matter" (Jo.Ke, 2015) is referred to in this paper as simply "Jo.Ke 1".

1.2 The paper "On the variation of the energy scale 2: galaxy rotation curves" (Jo.Ke, 2015) is referred to in this paper as simply "Jo.Ke 2".

1.3 The paper "On the variation of the energy scale 3: parameters for galaxy rotation curves" (Jo.Ke, 2015) is referred to in this paper as simply "Jo.Ke 3".

1.4 The rotation curves of many spiral galaxies remain flat in their outer regions and do not show the fall off in speed expected if the majority of the mass is concentrated in the galaxy centre. The widely accepted explanation for these observations is that galaxies are embedded in large haloes of dark matter.

1.5 The velocity dispersion of many clusters of galaxies show that the velocities of individual galaxies are far too high for the clusters to remain gravitationally bound. The clusters should have dispersed billions of years ago. Again the current explanation is that there must be large amounts of dark matter in the clusters holding the galaxies together.

1.6 'Jo.Ke 1' and 'Jo.Ke 2' showed that variations in the energy scale from location to location can explain the rotation curves of spiral galaxies. A simple model of: (a) a narrow Gaussian density distribution, and (b) a broader Gaussian for the energy scale variation gives good fits to the rotation curves of spiral galaxies.

1.7 This paper examines whether the same hypothesis can explain the velocities of galaxies in clusters of galaxies.

1.8 The problem is investigated in two ways:

(a) The expected velocities for circular orbits are calculated as a function of distance from the cluster centre.

(b) The escape velocity is examined by calculating the velocity a galaxy would have after free falling from the cluster boundary.

1.9 Both approaches show that higher velocities naturally arise than those expected for the observed cluster mass.

2 The cluster of galaxies model

2.1 We model a cluster of galaxies as a spherically symmetric sphere of gas with the galaxies embedded within it. Most of the mass is in the gas. We assume a single Gaussian density profile that covers both the gas and the galaxies. For the energy scale variation we assume a Gaussian with the same width.

2.2 The model is illustrated in Figure 1.

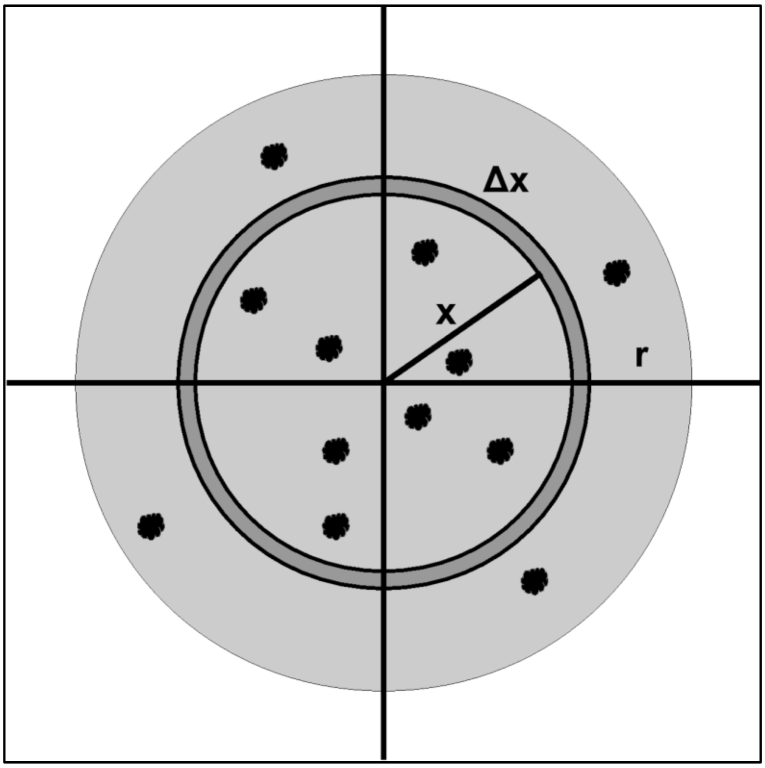

Figure 1. Model of a cluster of galaxies as a sphere of gas with galaxies embedded in it.

2.3 We assume a spherically symmetrical Gaussian density distribution given by:

$$\boldsymbol{\rho(x) = \rho_o \ exp(-x^2/\alpha^2)} \tag{1}$$

where $\boldsymbol{\rho_o}$ is the central density; $\boldsymbol{\alpha}$ is 1/e-width of the Gaussian; see Figure 2.

2.4 The mass of a spherical shell, of thickness $\boldsymbol{\Delta x}$, is given by:

$$\boldsymbol{\Delta M(x) = 4\,\pi\,x^2\,\rho_o\,exp(-x^2/\alpha^2)\,\Delta x} \tag{2}$$

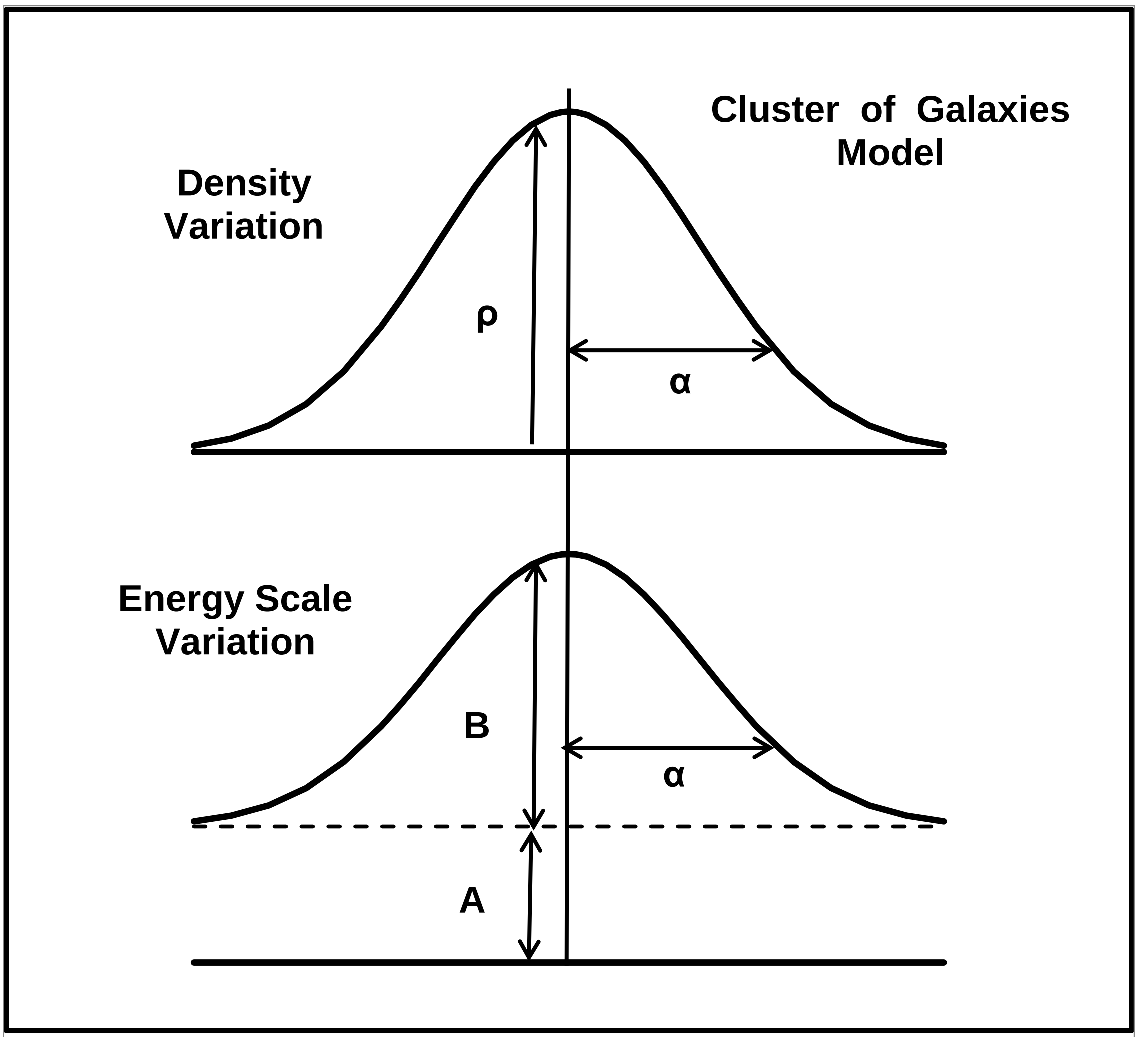

Figure 2. Gaussian profiles for the density and energy scale variations**.**

2.5 The total mass of the cluster of galaxies is given by integrating (2) from 0 to infinity

$$\boldsymbol{M = \sqrt{\pi}\,\pi\,\alpha^3\rho_0} \tag{3}$$

2.6 For convenience we set

$$\boldsymbol{z = x/\alpha} \tag{4}$$

and define

$$\boldsymbol{P(z) = z^2\,exp(-z^2)} \tag{5}$$

then the mass of a thin shell becomes

$$\boldsymbol{\Delta M(z) = \frac{4\,M}{\sqrt{\pi}}\;P(z)\,\Delta z} \tag{6}$$

2.7 Following 'Jo.Ke 1' and 'Jo.Ke 2' we assume the energy scale variation is given by:

$$\boldsymbol{\xi(x) = A + B\ \ exp(-\,x^2/\alpha^2\,)} \tag{7}$$

where for any given variation $\boldsymbol{A}$ and $\boldsymbol{B}$ are dimensionless constants; $\boldsymbol{\alpha}$ is the 1/e-width of the variation; see Figure 2.

2.8 We set

$$\boldsymbol{Q(z) = \ 1 + \ \gamma\ exp(-z^2)} \tag{8}$$

where

$$\boldsymbol{\gamma = \ B/A} \tag{9}$$

and $\boldsymbol{z{=}x/\alpha}$ (as before).

2.9 Following 'Jo.Ke 1' the mass of a thin shell at location $\boldsymbol{X}$ as measured by a galaxy (observer) at location $\boldsymbol{R}$ is

$$\boldsymbol{\Delta M_{RX} = \frac{Q(x)}{Q(r)}\ \Delta M_{XX} = \ \frac{Q(x)}{Q(r)}\ \Delta M(x)} \tag{10}$$

2.10 We set

$$\boldsymbol{y = \ r/\alpha} \tag{11}$$

and finally have

$$\boldsymbol{\Delta M_{RX} = \ \frac{4\ M}{\sqrt{\pi}}\ \ \frac{Q(z)}{Q(y)}\ P(z)\ \Delta z} \tag{12}$$

2.11 Equation (12) shows how the variation in the energy scale changes the effective mass of a shell of material at remote position $\boldsymbol{X}$ on a galaxy at $\boldsymbol{R}$. The difference is clear when comparing equation (12) with equation (6).

4 Velocities for circular orbits

4.1 We use the model described above to calculate the expected velocities for galaxies orbiting the cluster centre in circular orbits.

4.2 The usual velocity law for Keplerian orbits is

$$\boldsymbol{v^2 = \frac{G\,M(r)}{r}} \tag{13}$$

where $\boldsymbol{M(r)}$ is the mass interior to $\boldsymbol{r}$.

4.3 For our Gaussian density distribution and Gaussian energy scale variation we have instead, using equation (12)

$$\boldsymbol{v^2 = \frac{4\,G\,M}{\sqrt{\pi}\,\alpha}\ \frac{1}{y}\ \frac{1}{Q(y)}\int_0^y P(z)\,Q(z)\,dz} \tag{14}$$

4.4 The Keplerian velocity (i.e. no energy scale variation) is obtained by simply setting

$$\boldsymbol{Q(y) = Q(z) = 1} \tag{15}$$

which is equivalent to $\boldsymbol{\gamma = 0}$ in equation (8).

4.5 Figure 3 shows the orbital velocities for different energy scale variations, i.e. different values of $\boldsymbol{\gamma}$. The normalised velocities are obtained by setting

$$\boldsymbol{\frac{4\,G\,M}{\sqrt{\pi}\,\alpha} = 1} \tag{16}$$

in equation (14).

4.6 Approximately half of the cluster mass is within normalised distance $\boldsymbol{\alpha=1}$. It is apparent from Figure 3 that energy scale variations make no difference to the inner core of the cluster. However, higher than expected velocities are predicted in the outer parts of the cluster where, of course, there are fewer galaxies.

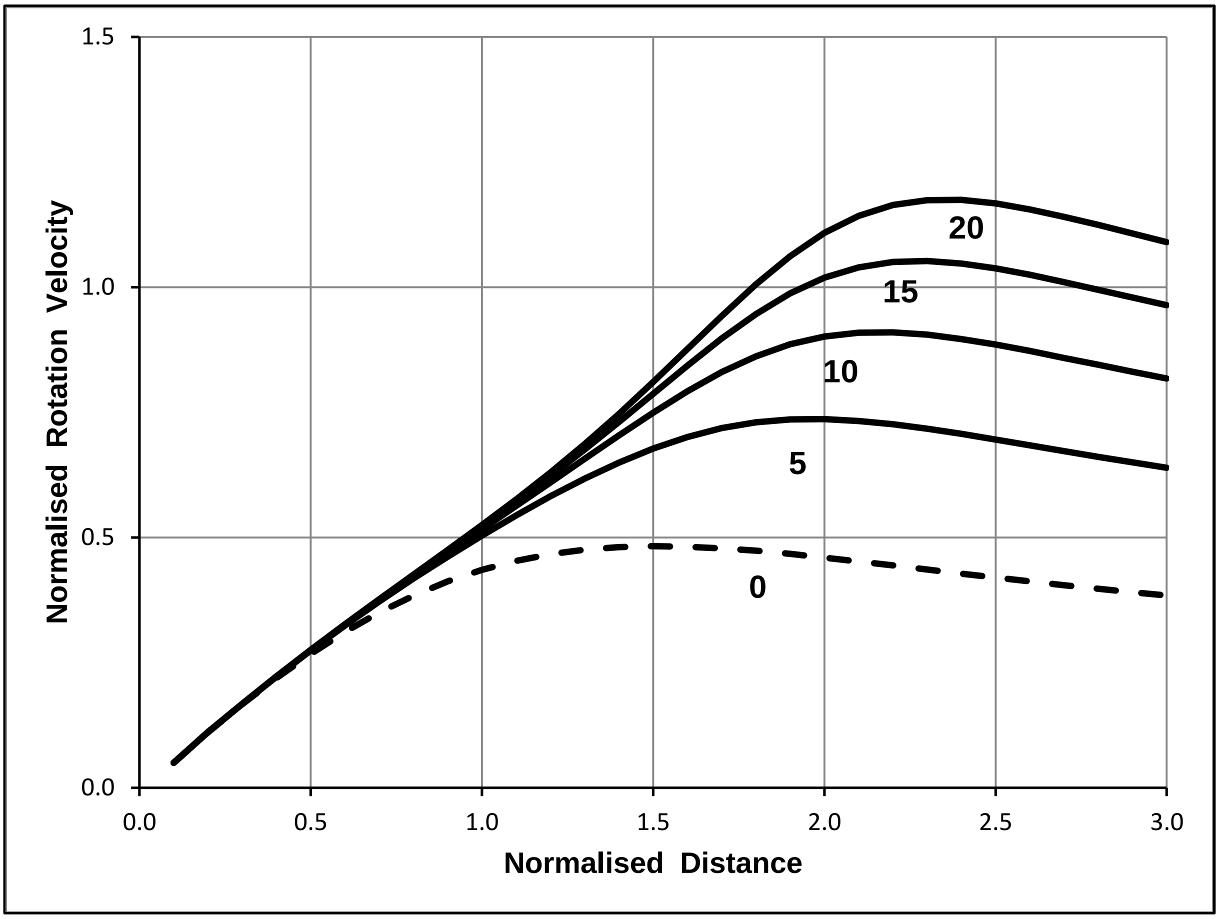

Figure 3. Rotation velocities calculated from equation (14) for different values, γ, of the energy scale variation as defined by equation (8). The dashed line is the curve for Keplerian orbits.

5 Escape velocities

5.1 The peculiar velocities of galaxies in clusters are too large for the observed cluster masses and the clusters should have dispersed in less than a billion years. We can investigate this by examining how variations in the energy scale affect the escape velocity.

5.2 Rather than calculate the escape velocity, we calculate the velocity a galaxy would have if it started from rest at 3α and underwent free-fall towards the cluster centre.

5.3 The acceleration for Newtonian gravity is

$$A(r) = -\frac{G\,M(r)}{r^2} \tag{17}$$

where $M(r)$ is the mass interior to r.

5.4 For our Gaussian density distribution and Gaussian energy scale variation we have instead, using equation (12):

$$A(r) = -\frac{4\,G\,M}{\sqrt{\pi}}\,\frac{1}{r^2}\,\frac{1}{Q(r)}\int_0^{r/\alpha} P(z)\,Q(z)\,dz \tag{18}$$

5.5 We define by

$$S(r) = \int_0^{r/\alpha} P(z)\,Q(z)\,dz \tag{19}$$

5.6 The work done by the Newtonian acceleration in moving a galaxy from $r=3\alpha$ to r, or alternatively from $y=3$ to $y=r/\alpha$ is converted into the galaxy's velocity

$$W(r) = \int_{3\alpha}^{r} A(r)\,dr = \frac{v^2}{2} \tag{20}$$

or

$$\frac{v^2}{2} = -\frac{4\,G\,M}{\sqrt{\pi}\;\alpha}\int_3^{r/\alpha}\frac{1}{y^2}\,\frac{S(y)}{Q(y)}\,dy \tag{21}$$

5.7 Figure 4 shows the 'escape' velocities for different values of the energy scale parameter γ. The normalisation is the same as for Figure 3, as in equation (16).

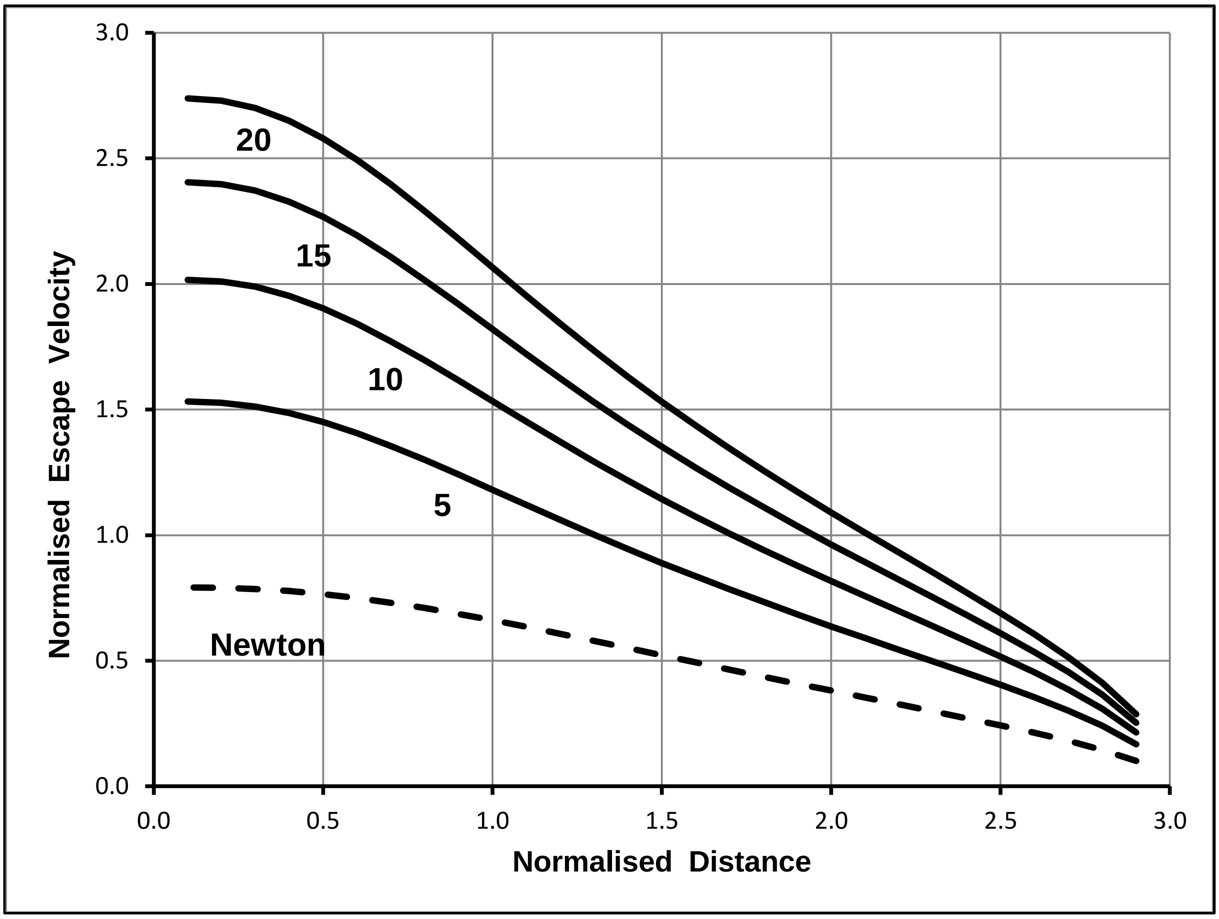

Figure 4. Escape velocities calculated from equation (21) for different values, γ, of the energy scale variation as defined by equation (8). The dashed line is the curve for Newtonian gravity, i.e. no energy scale variation.

γ	**velocity α=1**	**mass factor**		**velocity α=2**	**mass factor**
0	0.66	1.0		0.38	1.0
5	1.18	3.2		0.64	2.8
10	1.53	5.4		0.82	4.7
15	1.82	7.6		0.96	6.4
20	2.07	9.8		1.10	8.4

Table 1. Velocities taken from Figure 4. The 'mass factor' is how much missing mass can be accounted for.

5.8 The curves in Figure 4 show how, for a given cluster mass, different values of the energy scale variation give rise to higher than expected velocities for the galaxies.

5.9 The values of the escape velocities from Figure 4 are put together in Table 1 for two distances ($\alpha=1.0; \alpha=2.0$). The 'mass factor' column shows how much missing mass can be accounted for by variations in the energy scale.

5.10 For example, with $\gamma=10$ a galaxy at normalised distance $\alpha=1.0$ would have a normalised velocity of 1.53 compared to just 0.66 for pure Newtonian gravity. As the mass depends on velocity squared this results in a derived mass 5.4 times greater than the actual mass present.

5.11 Current estimates for the amount of missing matter in clusters of galaxies range from a factor of 4 to a factor of 9. These factors are covered by the variations in the energy scale shown in Figure 4 and enumerated in Table 1.

6 Virial theorem

6.1 Many studies of clusters of galaxies have used the observed velocities of the galaxies to estimate the mass of the cluster by applying the virial theorem.

6.2 For clusters of galaxies the virial theorem relates the average velocity to a measure of the linear size.

6.3 The virial theorem can be expressed as

$$< v^2 > = \frac{G\ M}{2\ < R >} \tag{22}$$

where $< v^2 >$ is the average velocity squared and $< R >$ is the linear size.

6.4 Equation (21) has a similar form to equation (22), and can be rewritten as

$$v^2 = K\ \frac{G\ M}{2\ \alpha} \tag{23}$$

where

$$K = -\frac{16}{\sqrt{\pi}} \int_3^{r/\alpha} \frac{1}{y^2} \frac{S(y)}{Q(y)}\, dy \tag{24}$$

6.5 We work with the median point of our model. Half the mass is interior to the distance $\alpha=1.1$, so we use that as our linear distance. We also use the escape velocity at $\alpha=1.1$ as our average velocity.

6.6 Table 2 lists the K values calculated from equation (24) for different values of γ, the size of the variation in the energy scale.

6.7 K can be considered as a correction to the virial mass of the cluster. It is clear from Table 2 that the observed missing mass factors of between 4 and 9 are covered by the values between 6 and 18.

6.8 This simple approximation to the virial theorem demonstrates how variations in the energy scale can explain the high velocities of galaxies in clusters of galaxies.

γ	K
0	1.0
2	1.9
4	2.8
6	3.6
8	4.5
10	5.4
12	6.2
14	7.1
16	8.0
18	8.8
20	9.7
22	10.6

Table 2. K factors for different values of γ, the size of the energy scale variation. K is the factor by which the mass of the cluster of galaxies must be increased over the mass given by the virial theorem.

7 Comments

7.1 It is clear that variations in the energy scale are capable of explaining the high velocities of galaxies in clusters of galaxies. There is no need to invoke the existence of dark matter.

7.2 We do not have ready access to scientific journals and have found it difficult to track down the data on clusters of galaxies. Measurements of average velocity, size, and derived mass could easily have been checked against the model to see how well they fit.

7.3 Current estimates for the missing mass in clusters of galaxies suggest a factor of 5.5, well within the range of values set out in Table 1 and Table 2.

7.4 The γ values for the size of the energy scale variation are similar to those used for the rotation curves of spiral galaxies, as set out in 'Jo.Ke 3'.

8 Conclusion

8.1 We apply the hypothesis of variations in the energy scale to clusters of galaxies. It is clear that the hypothesis can explain the anomalously high velocities of galaxies within the cluster.

8.2 There is no need to modify Newton's law of gravitation. There is no need to introduce any dark matter.

9 References

Jo.Ke 1. (2015). "On the variation of the energy scale: an alternative to dark matter".

Jo.Ke 2. (2015). "On the variation of the energy scale 2: galaxy rotation curves".

Jo.Ke 3. (2015). "On the variation of the energy scale 3: parameters for galaxy rotation curves".