On the variation of the energy scale 33

A Note on Hubble's Law

by

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Summary

Hubble's Law, with its linear relationship between velocity and distance, is often used to support the idea of an isotropic homogeneous Universe. However, a non-isotropic non-homogeneous Universe, made up of regions having random sizes and random expansion speeds, will also show a Hubble-type Law, with a linear relationship between velocity and distance.

1 Introduction

1.1 Hubble's Law is the observed linear relationship between the distance and redshift of distant galaxies. When the redshift is interpreted as a speed of recession, we get the distance-velocity relation, usually expressed as

$$\boldsymbol{V} = \boldsymbol{H_o}\,\boldsymbol{D} \qquad (1)$$

where V is the velocity; D the distance; H_o the Hubble constant (about 68 km/s/Mpc)

Hubble's law is described well by the Wikipedia article "Hubble's law" together with the peer-reviewed references cited therein.

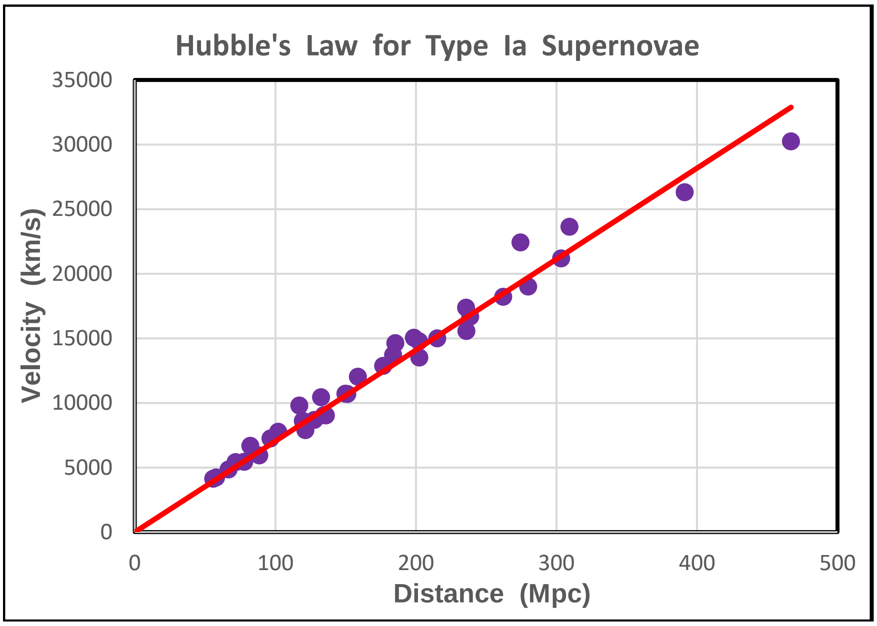

Figure 1. The Hubble diagram for type Ia supernovae (Freeman et al; 2001)

1.2 Figure 1 shows the Hubble diagram for type Ia supernovae in distant galaxies (Freeman et al, 2001), based on observations with the Hubble Space Telescope. The horizontal axis is the distance in Mpc. The vertical axis is the recession speed in km/s. The most distant supernovae are receding at around 30,000 km/s, which is 10% of the speed of light or a redshift of z=0.1. The Hubble diagram begins to depart from linearity for more remote objects.

1.3 The Hubble diagram provides strong evidence that the Universe is expanding. This is explained by the ΛCDM model of cosmology (Λ: the cosmological constant; CDM: cold dark matter), which is described well by the Wikipedia article "Lambda-CDM" together with the peer-reviewed references cited therein. The ΛCDM model assumes that the geometry of space is both isotropic and homogeneous, and separately that the Universe has a uniform energy density. Overall this means the Universe is smooth, and not made up of an aggregate of separate regions, all with their own expansion speeds.

1.4 In this note we show that a non-homogeneous and non-isotropic space also gives rise to a linear relationship between velocity and distance. So, the Universe can be clumpy rather than smooth, and made up of distinct regions with distinct expansion speeds.

2. A simple case

2.1 In this section we consider the simple situation of a line of-sight passing through regions with different sizes and expanding with different speeds. This is illustrated in Figure 2 below. What distance-velocity relation do we expect to observe?

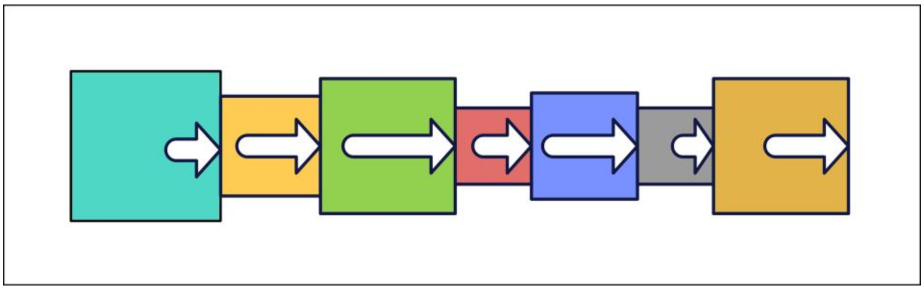

Figure 2. Universe made of regions of different sizes and different expansion speeds. The rectangles represent the individual regions and the arrows the individual expansion speeds.

2.2 The distance to the far side of the kth region, D_k ,is simply the sum of the sizes of all the regions out to region k

$$D_k = \sum_{j=1}^{k} S_j \tag{2}$$

where S_j is the size of the jth region.

If $\overline{S}$ is the average size of a region, then equation (2) can be written as

$$D_k = \sum_{j=1}^{k} \overline{S} + \sum_{j=1}^{k} (S_j - \overline{S}) \tag{3}$$

where the second summation is simply the difference in size from the mean. If the regions have random sizes, then there are as many regions larger than the mean as smaller than the average. So, for large k, the second summation in equation (3) tends to zero, and we are left with

$$D_k \approx \sum_{j=1}^{k} \overline{S} = k\,\overline{S} \tag{4}$$

2.3 The expansion speed of the far side of the kth region, V_k , is simply the sum of all the expansion speeds out to region k

$$V_k = \sum_{j=1}^{k} U_j \tag{5}$$

where U_j is the expansion speed of the jth region.

If $\overline{U}$ is the average speed of a region, then equation (5) can be written as

$$V_k = \sum_{j=1}^{k} \overline{U} + \sum_{j=1}^{k} \left(U_j - \overline{U} \right) \tag{6}$$

where the second summation is simply the difference in the speed from the mean. If the regions have random expansion speeds, then there are as many regions expanding faster than the average as slower than the mean. So, for large k, the second summation in equation (6) tends to zero, and we are left with

$$V_k \approx \sum_{j=1}^{k} \overline{U} = k\,\overline{U} \tag{7}$$

2.4 We can combine equations (4) and (7) to give

$$V_k = \left(\frac{\overline{U}}{\overline{S}} \right) D_k \tag{8}$$

The bracketed term is constant and so this equation is identical to Hubble's Law, equation (1). Consequently, as we move out towards large k, we expect to observe a linear relationship between velocity and distance.

2.5 Figure 3 below shows an example of the velocity-distance law for a Universe made up of separate regions. The regions have random sizes from 0 to 100 and random velocities from 0 to 25. The red line is the linear fit to the data. There is a clear linear relationship but with a fair amount of scatter.

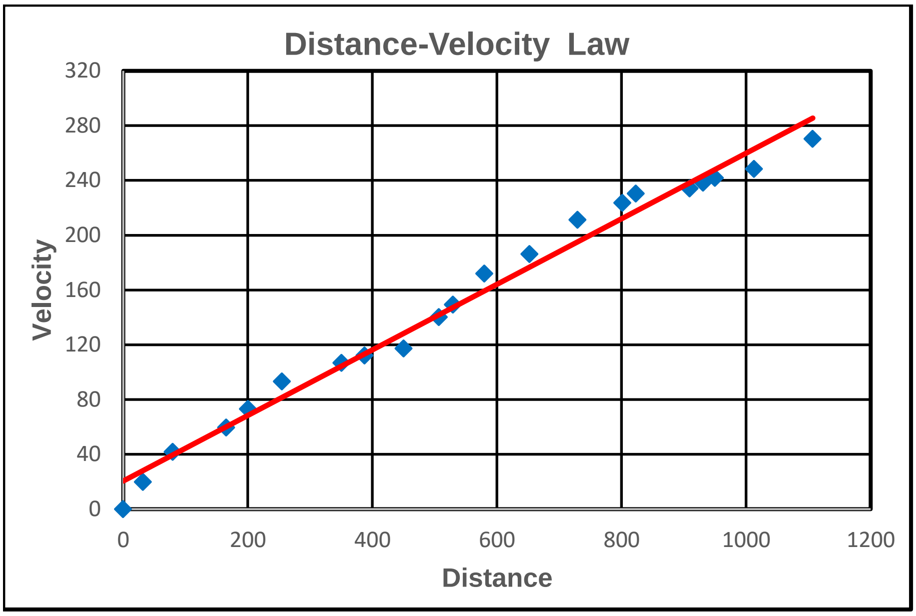

Figure 3. Distance-Velocity diagram for a Universe made of regions with random sizes from 0 to 100 and random velocities from 0 to 25.

3. A more realistic case

3.1 The real Universe is somewhat more complicated than the simple case discussed above. We consider two complications:

(a) the Universe is old and the regions have expanded over time,

(b) the line of sight generally intersects with only a fraction of each region.

3.2 We can construct a more realistic picture by starting with regions of different sizes and velocities, and allowing the regions to expand over time. And we can include only a fraction of each region to mimic the fact that only a fraction of each region lies along the line of sight. This situation is illustrated in Figure 4 below, with the expansion velocities omitted.

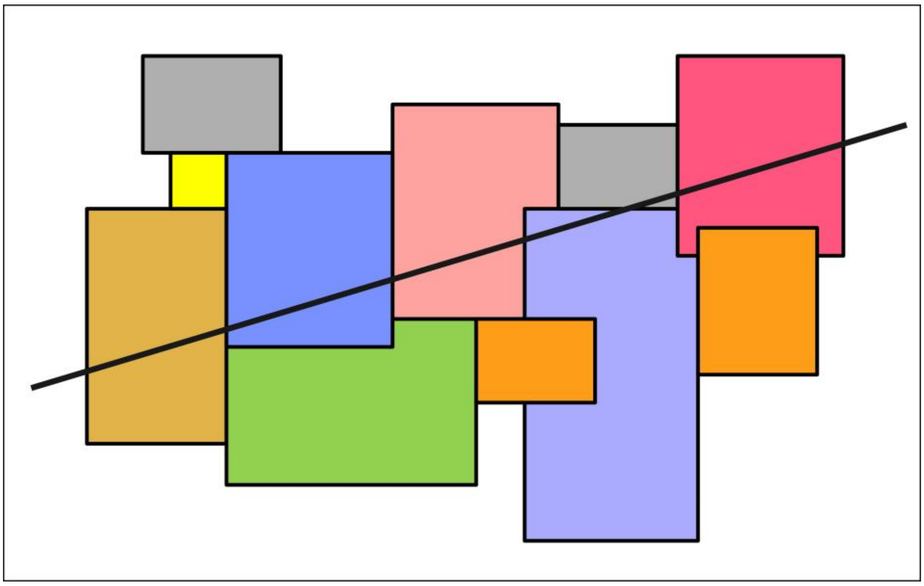

Figure 4. The solid line represents the line-of-sight passing through different regions, which have their own individual expansion speeds.

3.3 For the jth region, the line-of-sight only passes through a fraction, $\boldsymbol{f_j}$, of the region, where $0 \leq f_j \leq 1$. So, the distance traversed is $f_j \, S_j$. The expansion speed is also reduced by the same fraction to $f_j \;\; U_j$.

The distance to the far side of the kth region is now given by

$$\boldsymbol{D_k} \;=\; \sum_{\boldsymbol{j=1}}^{\boldsymbol{k}} \left(\boldsymbol{f_j \, S_j} \;+\; \boldsymbol{f_j \, U_j \, t} \right) \qquad (9)$$

In the summation the first term is the initial size; the second term is the expansion over time $\boldsymbol{t}$. For large times, $\boldsymbol{t}$, the second term in the summation becomes dominant and, ignoring the first term, we have

$$\boldsymbol{D_k} \approx \boldsymbol{t} \sum_{j=1}^{k} (\boldsymbol{f_j}\, \boldsymbol{U_j}) \quad (10)$$

The expansion speed of the far side of the kth region is given by summing the individual expansion speeds

$$\boldsymbol{V_k} = \sum_{j=1}^{k} (\boldsymbol{f_j}\, \boldsymbol{U_j}) \quad (11)$$

3.4 Combining equations (10) and (11) we have

$$\boldsymbol{V_k} = \frac{\boldsymbol{1}}{\boldsymbol{t}}\, \boldsymbol{D_k} \quad (12)$$

which for a given time, $\boldsymbol{T}$, is

$$\boldsymbol{V_k} = \left(\frac{\boldsymbol{1}}{\boldsymbol{T}}\right) \boldsymbol{D_k} \quad (13)$$

The bracketed term is a constant (fixed $\boldsymbol{T}$) and we are back to Hubble's Law with a linear relationship between velocity and distance. $\boldsymbol{T}$ is interpreted as being the age of the Universe and also the reciprocal of the Hubble constant. Equation (13) only holds for large times $\boldsymbol{T}$, which is fine if we are talking about the age of the Universe.

3.5 Figure 5 below shows the starting velocity-distance relation for a lumpy Universe made up of separate regions with different expansion speeds. The regions have random sizes from 0 to 100 units, random expansion speeds from 0 to 10 units, and line-of-sight intersections of a random fraction between 0.0 and 1.0. There is an approximately linear relationship between velocity and distance, but with considerable scatter.

3.6 Figure 6 below shows that position after 10 timesteps. We have kept the velocities the same, but allowed the Universe to expand and more than double in size. There is a much stronger linear relationship as the velocities start to wash out the difference is the sizes of the regions.

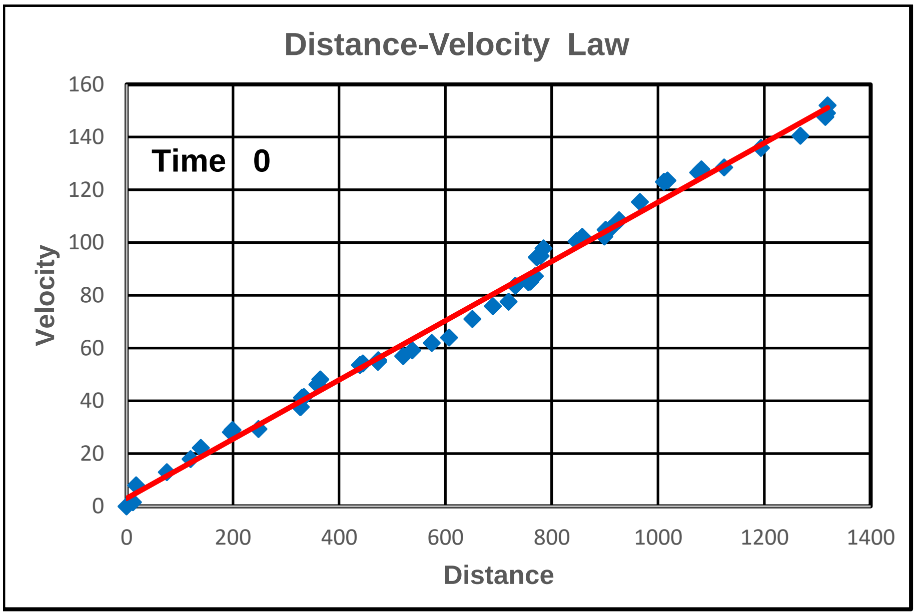

Figure 5. Example of a Universe made up of separate regions. The regions have random sizes from 0 to 100 and random velocities from 0 to 10. This diagram shows the initial position at timestep 0.

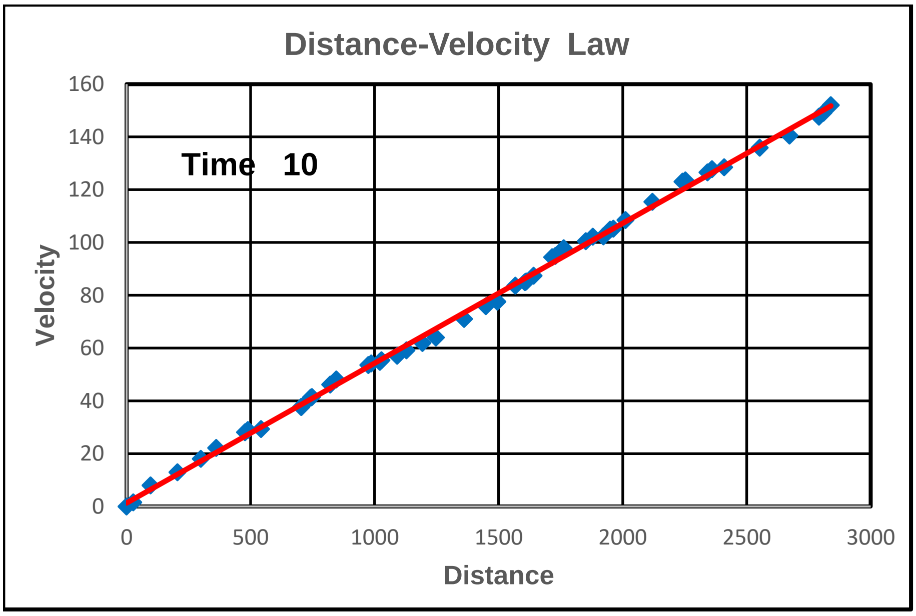

Figure 6. Same as Figure 5, but at timestep 10 when the Universe has increased in size to around 2800 units. The scatter of data points is reduced and the relation is becoming more linear.

3.7 Figure 7 below shows the Universe after 100 timesteps, when it has expanded to around 13 times its original size. There is hardly any scatter and the velocity-distance relation is getting very close to being a perfect straight line. One might conclude that the Universe has become smooth but the separate regions still exist and they still have their separate expansion speeds. So, the Universe is still lumpy.

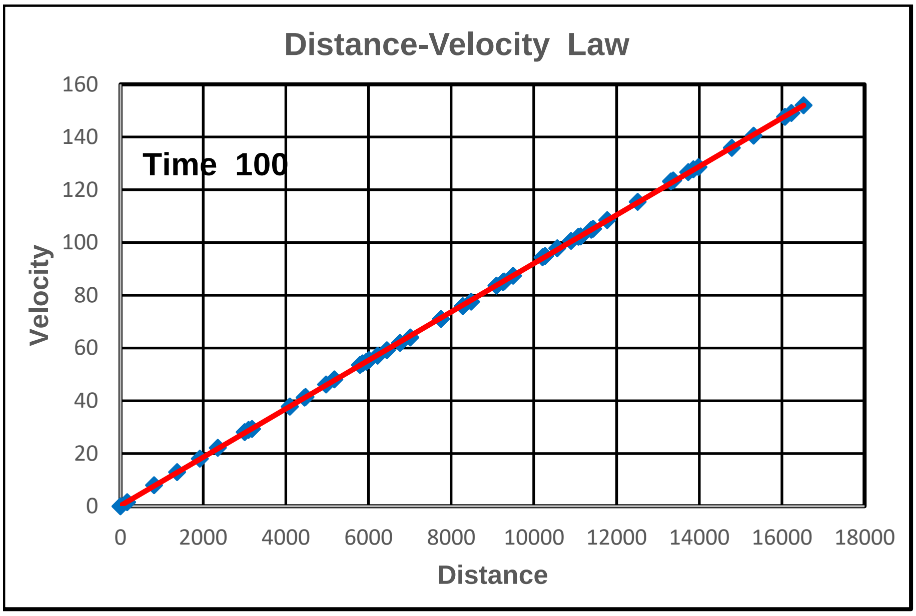

Figure 7. Same as Figure 5, but at timestep 100 when the Universe has increased in size to around 17,000 units. The scatter of the data points is tiny giving a near perfect straight line.

3.8 The real Universe has expanded by a factor of around 1100 since the time of the cosmic microwave background (CMB). This is much greater than the factor of 13 shown in Figure 7 above. So, we would expect a lumpy Universe to show a very good straight line at today's epoch.

4. Discussion

4.1 This note discusses the world map view of the Universe, i.e. how the Universe appears to a god-like entity looking down on it and seeing how it appears at any given instant. To obtain our human-based observational view, we need to take into account the light travel times and the expansion of the Universe. For this we need a cosmological model, such as the ΛCDM model. However, if we restrict ourselves to low redshift values ($z < 1.0$) and short lookback times, then the world map view is reasonably close to our observational view (Harrison, 2000). So, Figure 7 should still hold good.

4.2 The observed linear relationship between velocity and distance of the Hubble Law is strong evidence that the Universe is expanding. It is often used to support the Universe being isotropic and homogeneous. However, in this note we have demonstrated that a non-isotropic and non-homogeneous Universe will also have a linear relationship between velocity and distance. So, although an isotropic and homogeneous Universe will give rise to the Hubble Law, we cannot argue the other way round and use the Hubble Law to prove that the Universe is isotropic and homogeneous.

4.3 What we have presented here is a very simple result and must surely have been reported elsewhere. However, it does not appear to be mentioned in any of the standard works on cosmology.

4.4 No artificial intelligence (AI) was used in this work.

5. References

Freeman WL et al. 2001. Astrophysical Journal, **553**, 47.

Harrison, E. 2000. "Cosmology" 2nd edition. Cambridge University Press.

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Wikipedia article "Lambda-CDM model ".
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