On the variation
of the
energy scale 30

Disk Galaxies and
Galaxy Clusters

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Summary

We present the first linear relationship that explains both the rotation curves of disk galaxies and the dynamical masses of galaxy clusters, without the need for any dark matter. A previous paper analysed the rotation curves of galaxies from the SPARC catalogue and a linear relationship was found. Similar analysis has been applied to recently published data on galaxy clusters and the same linear relationship has been found. This linear relationship is an observational result, not a theoretical one. It rules out the requirement for any dark matter in disk galaxies and galaxy clusters.

Given the distribution of baryonic matter (stars, gas) in a spiral galaxy, we can use the linear relationship to predict the rotation curve; something dark matter cannot do. And given the distribution of baryonic matter (gas, galaxies) in a galaxy cluster, we can use the same linear relationship to predict the dynamical mass; again, something dark matter cannot do. The analysis is interpreted as meaning the energy scale varies from location to location. No dark matter is required, and no changes to Newtonian gravitation are required either; just a simple variation of the energy scale.

1   Introduction

1.1   Physicists love a straight line and are delighted whenever their experiments reveal a linear relationship between two quantities. Simple examples are (a) Ohms' Law for the voltage and current in an electrical circuit, and (b) Hooke's Law for the weight and extension of an elastic spring.
However, many quantities have a power law relationship and so do not show a straight line. For example, the volume of a cube is not linearly dependent on the length of its side. Here physicists apply a different trick and plot the logarithms of the quantities against one another. This trick often reveals a linear relationship. In the case of the cube, the logarithmic plot shows a straight line of slope 3. Logarithmic plots are what we use in the work presented here.

1.2   When astronomers look at the rotation curves of spiral galaxies, they often find the curves are flat in the outer regions, i.e., the velocity is constant away from the galaxy centre. This is in disagreement with Newtonian gravity where a drop off in velocity is expected.   The current way of explaining the observations is to postulate the existence of large amounts of dark matter in a spherical halo surrounding the galaxy. Given the distribution of baryonic matter across a spiral galaxy, astronomers cannot predict the shape of the rotation curve; they can only calculate the amount of dark matter that is required to explain the observed velocities (Sanders, 2010).   There are no linear relationships for the rotation curves of spiral galaxies.

1.3   When astronomers look at galaxy clusters, they can estimate the amount of baryonic matter by observing the individual galaxy members and by observing the X-ray emission from the hot gas. Such observations show that the galaxies account for around 10% of the baryonic mass; the gas supplying the other 90%. Separately, astronomers can observe the velocities of the galaxies and apply the virial theorem to obtain the so-called dynamical mass. For undisturbed clusters, they can assume the X-ray gas is in hydrostatic equilibrium and obtain a second estimate of the dynamical mass. And they can also observe the gravitational lensing of remote galaxies by the whole cluster and obtain a third estimate of the dynamical mass. The different measures of the dynamical mass are in good agreement with one another but are around five times greater than the baryonic mass. As with spiral galaxies, the current way of explaining this discrepancy is to assume galaxy clusters have their own haloes of dark matter (Sanders, 2010). There are no linear relationships for the dynamical masses of galaxy clusters.

1.4   No satisfactory answer has been found to the mass discrepancy observed in disk galaxies and galaxy clusters (and other scenarios), and the problem is the subject of intense research activity amongst both astronomers and physicists. Currently, there are two classes of solution,

(a)   dark matter. Some new form of non-baryonic matter exists beyond the particles in the standard model of particle physics. Example of such particles include: wimps; sterile neutrinos; axions. The matter discrepancy is solved by postulating the existence of one of these hypothetical particles and adding in sufficient quantities of them so that the problem goes away. Most astronomers and physicists believe that some form of dark matter exists.

(b)   modification of the law of gravity. There is no dark matter, and the observed mass discrepancy arises because our law of gravity is incomplete in the regimes of disk galaxies and galaxy clusters. The best known of these hypotheses is MOND (Modified Newtonian Dynamics), first suggest by Milgrom (Sanders, 2010). For MOND, the usual Newtonian formula applies in high acceleration regimes, and a modified formula applies in low acceleration regimes. The crossover acceleration is around 1.0×10^{-10} m.s^{-2}.

We offer a third solution, namely,

(c)   variation of the energy scale. The energy at one location as measured by an observer at a different location depends on the values of the energy scale at both locations. This idea also applies to the mass scale. It means the observed dynamical mass can differ from the observed baryonic mass.

1.5   The paper "On the variation of the energy scale: an alternative to dark matter" (Jo.Ke, 2015) first introduced the idea of variations of the energy scale to explain the rotation curves of spiral galaxies. That paper showed that the rotation curves of spiral galaxies could be explained by a simple variation of the energy scale and that no dark matter was needed. Follow up papers looked at other astronomical scenarios where dark matter was invoked and showed that energy scale variations had the potential to provide an alternative explanation.

1.6   The SPARC catalogue of spiral galaxies (Lelli et al, 2016) provided the rotation curves and the baryonic matter distributions for 175 galaxies. This enabled a deeper analysis to be carried out, which revealed a relationship between galactic distance and the ξ-function describing the variation of the energy scale. When plotted on a logarithmic scale an unexpected linear relationship was found. Without explicitly looking for it, we had stumbled across a linear relationship that explained the rotation curves of spiral galaxies. There was just one adjustable parameter, the slope of the straight line; a different slope was needed for each galaxy. This work is fully described in viXra paper 1903.0109 ("An analysis of the rotation curves of disk galaxies using the SPARC catalogue").

1.7   Recently measurements of the distribution of both baryonic mass and dynamical mass for a number of galaxy clusters have been published (Li et al, 2023). Five of these galaxy clusters are fully relaxed and undisturbed, and have data for individual galaxy members as well as X-ray data for the hot intracluster gas. These five galaxy clusters have been analysed in a similar manner to the SPARC galaxy and the same linear relationship has been found.

1.8   In section 2 "Variation of the energy scale" we re-examine what it means for the energy scale to vary from location to location. We extend this to variations of the mass scale and how these can explain the flat rotation curves of disk galaxies without the need for any dark matter. In section 3 " Disk galaxies" we revisit the data for two galaxies from the SPARC catalogue (Lelli et al), showing in particular the linear relationship. In section 4 "Galaxy clusters" we present new results for five galaxy clusters (Li et al, 2023) and show the linear relationship also holds there. In section 5 "A linear relationship" we look at the linear relationship found for both disk galaxies and galaxy clusters, and consider what this tells us about the nature of energy scale variations.

2   Variation of the energy scale

2.1   The conjecture that the energy scale might vary from location to location was put forward by Jo.Ke (JoKe, 2015), and developed in subsequent papers. The conjecture is fully explained in viXra paper 2007.0017 "Variation of the energy scale: an alternative to dark matter" (Jo.Ke, 2020). Assuming it is only the energy scale that varies and that the other physical scales (including length and time) are constant, then the conjecture also means that the mass scale can vary from location to location. This means the apparent dynamical mass of an object can be different from its baryonic mass, which leads to our alternative explanation for dark matter.

2.2   We must state at the outset that variation of the energy scale is pure conjecture. There is no evidence that it can happen or actually exists. The motivation for the conjecture is that it provides a coherent and alternative explanation to dark matter.

2.3   We start by introducing a scalar function of position ξ that describes the value of energy (mass) scale at every location in space and time. The effect of a mass at remote location P on local location X is then given by.

$$\xi_X \, M_P^X \;=\; \xi_P \, M_P^P \tag{1}$$

where ξ_X is the value of the ξ function at location X,

M_P^X is the mass at P as measured by an observer at X,

ξ_P is the value of the ξ function at location P,

M_P^P is the mass at P as measured by an observer P.

A subscript indicates the location of the object; a superscript indicates the location of the observer.

2.4   It can be seen that the conjecture only works because we are dealing with two different locations. For example, we have stars in the spiral arms of galaxies revolving about a massive galactic centre. The rotational velocity of a star in the spiral arms at location X is caused by the mass of the galaxy concentrated towards the centre at location P . If both masses are at the same location, then the ξ factors cancel out and there is no effect. This is why no variations of the energy scale can be detected on Earth or in the solar system.

2.5   In terms of baryonic and dynamical masses, it follows that M_P^P is the baryonic mass, and M_P^X is the dynamical mass. So, rearranging equation (1), the dynamical mass is given by.

$$M_P^X \;=\; \left(\frac{\xi_P}{\xi_X}\right) M_P^P \tag{2}$$

It is easy to see that a disk galaxy with central (baryonic) mass M_P^P, and a ξ-function that decreases away from the centre (i.e. $\xi_P > \xi_X$) will have a much larger dynamical mass M_P^X further out in the disk.

2.6   Consider the following hypothetical example. We have a central mass of 100 units and a ξ-function that decreases away from the centre. We look at what this means for massless test particles at various distances.
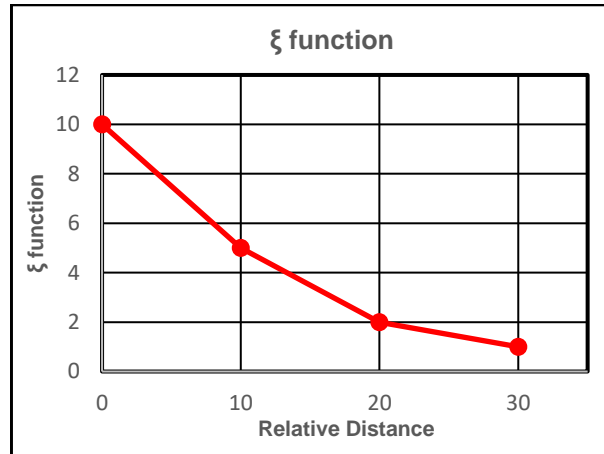

Figure 1. ξ-function. An arbitrarily set of values for the ξ-function, declining away from a central mass.

Figure 1 shows how the ξ-function varies with distance. We have chosen four arbitrary values so that the ξ-function decreases away from the central mass.
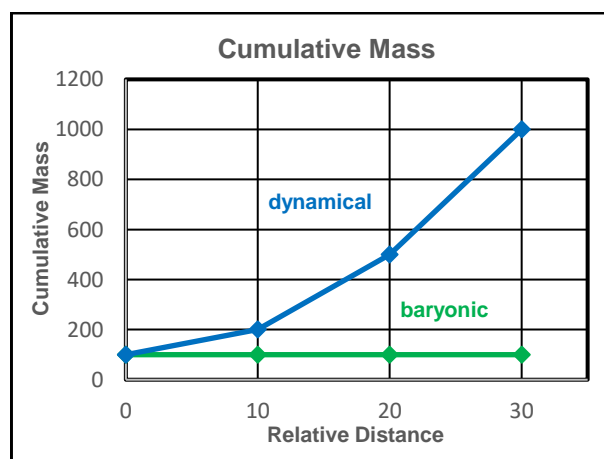
Figure 2. Cumulative mass. The green line is the cumulative baryonic mass as measured by a remote observer. The blue line is the cumulative dynamical mass as measured by a remote observer.

Figure 2 shows the cumulative mass as measured by a remote observer. The green line is the cumulative baryonic mass, which remains constant at 100 units. This follows because we are working with a central mass with no additional mass away from the centre. It corresponds to what observers would measure for galaxies based on photometric data and a mass-to-light ratio. The blue line is the cumulative dynamical mass, which increases away from the centre in accordance with equation (2). This corresponds to what observers deduce from the rotational velocities of disk galaxies and from the velocities of galaxy members of a galaxy cluster. It is seen that even though there is no additional mass away from the centre, a variation in the energy scale can give rise to an increasing dynamical mass.

Figure 2 is illustrative of diagrams often presented for galaxy clusters.
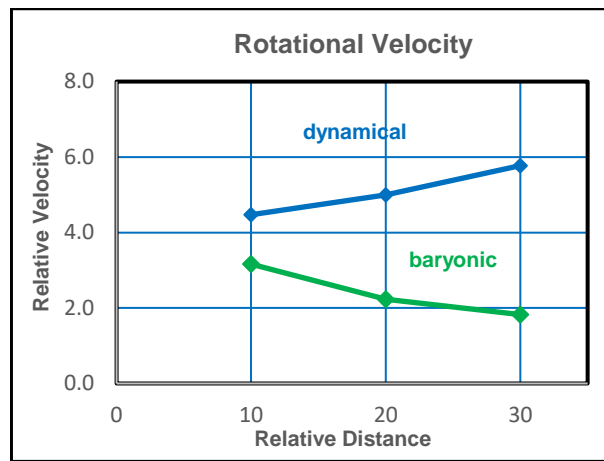
Figure 3. Rotation Curve. The green line (baryonic) is the expected curve for a central mass and Newtonian gravity. The blue line (dynamical) is what observers see with a variation of the energy scale.

Figure 3 shows the rotation curve for our hypothetical values. The relative velocities have been calculated from the usual Newtonian formula

$$v = \sqrt{\frac{G\,M}{r}}$$

(3)

and setting G=1. The green line show the usual Newtonian decline expected for a central mass; it is based on equation (3) where the mass is the baryonic mass. The blue line is what observers would see for massless test particles in a disk galaxy; it is based on equation (3) where the mass is the dynamical mass. This figure shows how a variation of the energy scale can give rise to rotational velocities that are much higher than the expected velocities.

Figure 3 is illustrative of diagrams often presented for disk galaxies.

Distance	ξ	baryonic mass	dynamical mass	baryonic velocity	dynamical velocity
0	10	100	100		
10	5	100	200	3.2	4.5
20	2	100	500	2.2	5.0
30	1	100	1000	1.8	5.8

Table 1. The hypothetical values used in Figures 1, 2 & 3.

2.7   For a disk galaxy, where the density falls off exponentially, we can assume the mass interior to a given radius is concentrated at the centre without incurring any significant errors (Binney & Tremaine, 2010?). The rotational velocity, $v(r)$, at distance r is then given by.

$$v(r)^2 \; = \; \frac{G}{r \; \xi_r} \int_0^r \xi_X \, dM_X^X \qquad\qquad (3)$$

where ξ_r is the value of the ξ-function at r ; ξ_X is the value of the ξ-function at X; dM_X^X is the mass of the incremental shell at X. So each incremental shell is weighted by the local value of ξ, and the whole integral is then divided by the value of ξ at r.

This formula is used for explaining the rotation curves of disk galaxies as covered in the next section.

2.8   For a galaxy cluster, which is spherical in shape, the total dynamical mass interior to radius r is given in terms of the baryonic mass by

$$M(r)_{\mathrm{dyn}} \; = \; M_r^r \; = \; \frac{1}{\xi_r} \int_0^r \xi_X \, dM_X^X \qquad\qquad (4)$$

where M_r^r is the mass interior to r as measured by an observer at r (i.e. the dynamical mass); ξ_X is the value of the ξ-function at X; dM_X^X is the mass of the incremental spherical shell at X (i.e. the baryonic mass).

This formula is used for explaining the relationship between the baryonic and dynamical masses of galaxy clusters.

3　Disk Galaxies

3.1　Mass models and rotation curves for 175 of disk clusters have been published by Lelli et al (2016). 70 of these were analysed in viXra paper 1903.0109 (Jo.Ke., 2019); the remainder were omitted because they had too few data points or only covered the central regions. The dynamical masses come from the observed rotation curves. The baryonic masses come from photometric observations in the infrared and radio wavelengths.
The data on 4 galaxies are presented in viXra paper 1903.0109.
The data on an additional 64 galaxies are presented in "SPARC galaxy rotation curves" (Jo.Ke., 2019).
The following pages show the data for just two galaxies; the data for the other galaxies are available at the above locations.

3.2　The upper left panel shows the rotation curves using the observed data of Lelli et al (2016). The black diamonds are observed velocities. The purple curve is the contribution to the velocity from the central bulge (if one exists); the orange curve from the disk of stars; the green curve from the gas. The blue curve is the expected velocity given by aggregating the other components.

3.3　The top right panel shows the cumulative mass distribution corresponding to the velocities in the top left panel. The black diamonds give the observed total mass corresponding to the black diamonds in the top left panel. The blue line gives the normal matter mass corresponding to the blue line in the top left panel. This diagram shows whether the observed or expected masses are levelling off or are still increasing at the outer edge of the galaxy. The observed mass usually shows a continuing increase; the expected mass usually shows convergence.

3.4　The lower left panel is a logarithmic plot of the ξ-function against the radial distance. The black diamonds are the values of the ξ-function and are based solely on the observed dynamical and baryonic masses. The near linear relationship away from the cluster centre is very clear. The red line is a straight line fit to the data, ignoring the first few data points. The linear relationship came as a surprise; it was completely unexpected.

3.5　The bottom right panel shows the rotation curve again. The black diamonds are the same observed velocities as in the top left panel. Similarly, the blue line is the same expected velocities as in the top left panel. The red line is the fitted rotation curve derived by applying the red line from the bottom left panel for ξ-function to the blue line from the top right panel.

Disk Galaxy NGC 2403
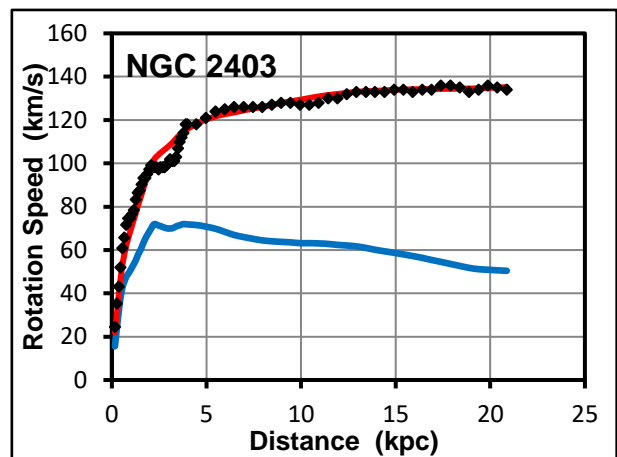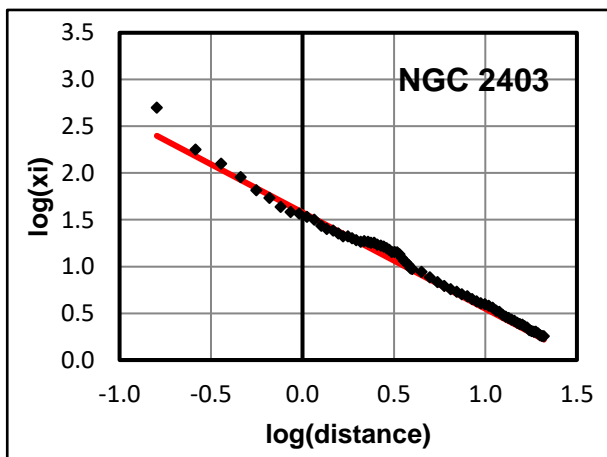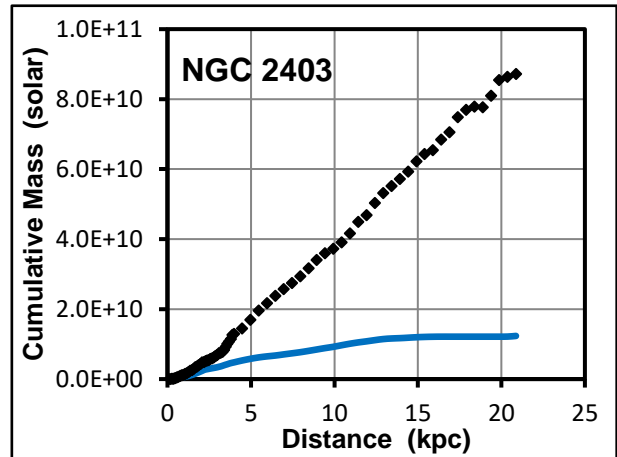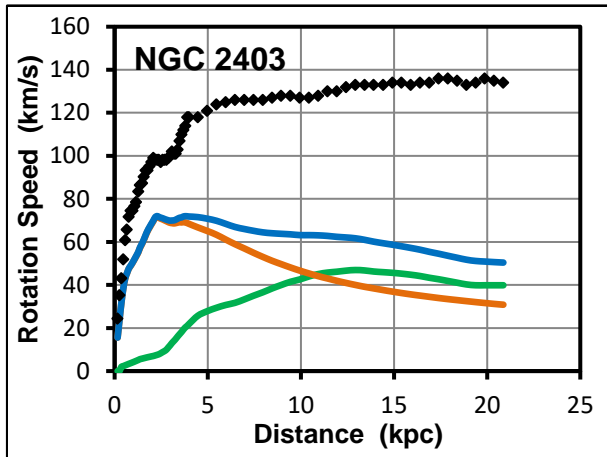
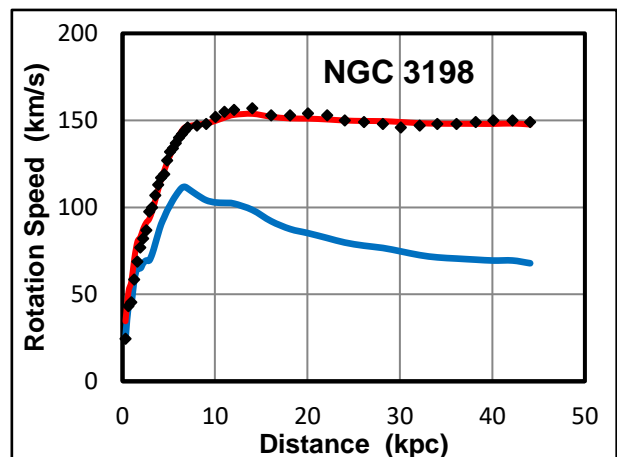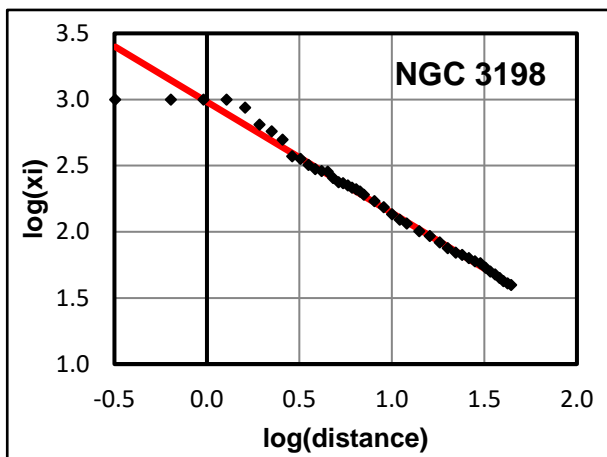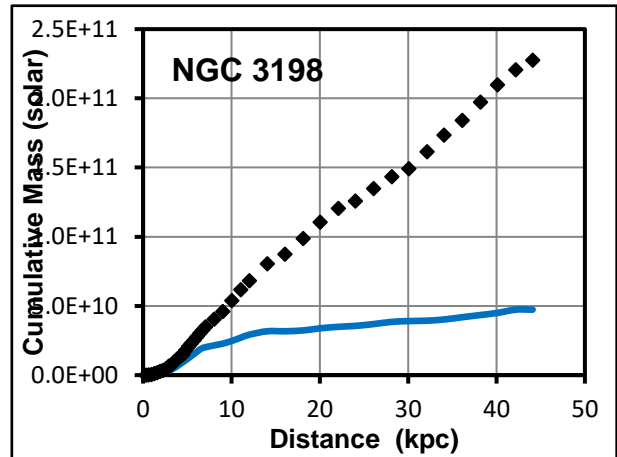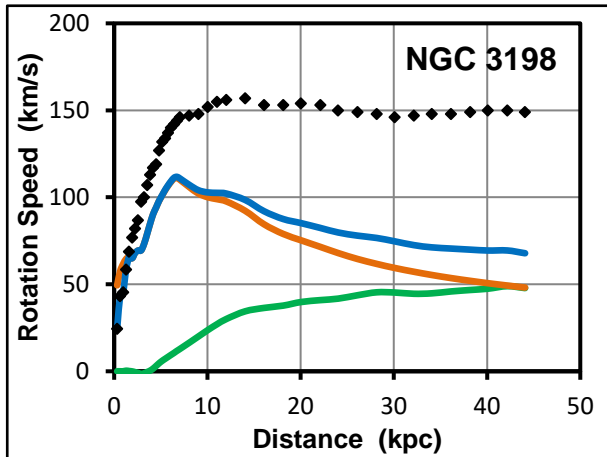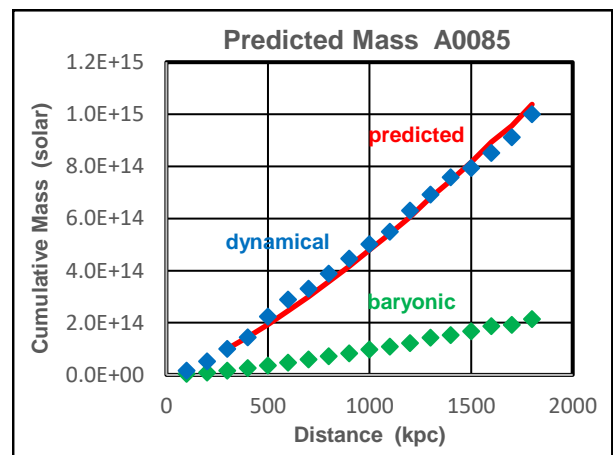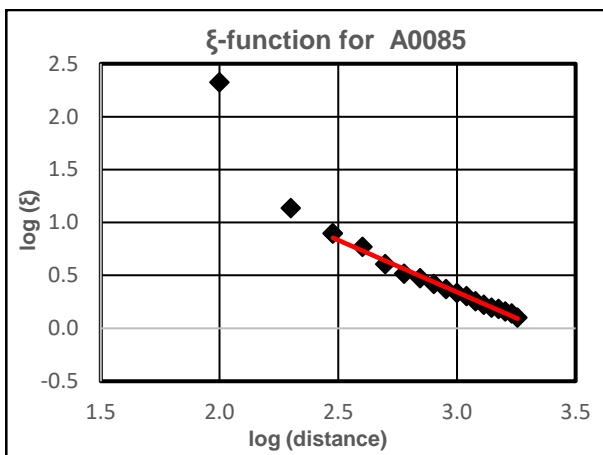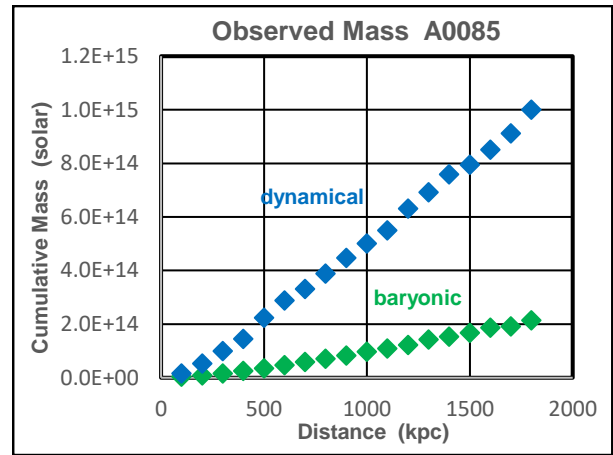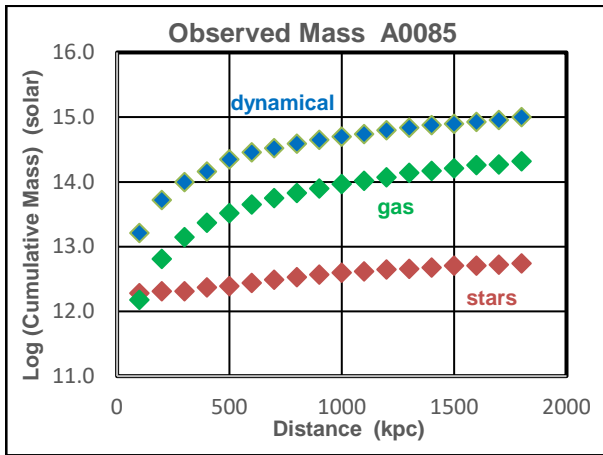
Slope of ξ-function:　　　-1.03

The red curve in the bottom right panel is the predicted shape of the rotation curve. It is a good fit to the observed rotation curve (black diamonds). It is based on the baryonic mass distribution (blue curve) and a straight line for the ξ-function, similar to that shown in the bottom left panel.

The upper right panel shows that the total baryonic mass (blue line) of the galaxy has converged by 15 kpc, whereas the total dynamical mass (black diamonds) continues to increase.

This galaxy is used as a typical disk galaxy throughout the book "The Dark Matter Problem" (Sanders, 2010).

Disk Galaxy NGC 3198

Slope of ξ-function: -0.84

The red curve in the bottom right panel is the predicted shape of the rotation curve. It is a good fit to the observed rotation curve (black diamonds). It is based on the baryonic mass distribution (blue curve) and a straight line for the ξ-function, similar to that shown in the bottom left panel.

The upper right panel suggests that the total bayronic mass (blue line) has converged just after 40 kpc, whereas the total dynamical mass (black diamonds) continues to increase and shows no sign of levelling out.

4.   Galaxy Clusters

4.1   Data on the baryonic and dynamical masses for a number of galaxy clusters have been published by Li et al (2023). We have analysed five of these clusters, omitting those that are disturbed and those with no X-ray data. The dynamical masses come from the velocity dispersion of galaxy members and the hydrostatic mass of the gas. The baryonic masses come from the X-ray gas, surface brightness fits, and the galaxy masses.

4.2   The upper left panel shows the data taken from Li et al (2023). The blue diamonds are the dynamical mass; green diamonds the mass of gas; orange diamonds the mass of stars. The dynamical mass is the average of the different values (velocity dispersion, hydrostatic mass, surface brightness). The gas mass is the average of the different values (gas mass profiles, surface brightness). The data comes as a logarithmic plot, which was measured manually. We did not have access to the linear (non-logarithmic) values or any tabular data.

4.3   The upper right panel is identical to the upper left diagram but with the data plotted on a linear scale. The blue diamonds are the dynamical masses; the green diamonds the baryonic masses. The baryonic mass is the sum of the gas and the stars, as shown in the upper left figure.

4.4   The lower left panel is a logarithmic plot of the ξ-function against the radial distance. The black diamonds are the values of the ξ-function and are based solely on the observed dynamical and baryonic masses. The near linear relationship away from the cluster centre is very clear. The red line is a straight line fit to the data, ignoring the first two data points. This is an observational result based on data from Li et al (2023) and the assumption of variations of the energy scale. The linear relationship came as a surprise; it was completely unexpected.

4.5   The lower right panel is identical to the upper right diagram. The solid red line is the predicted dynamical mass based on the observed baryonic and a linear ξ-function similar to that shown in the lower left diagram. It is clear in all cases that the predicted dynamical mass (red line) is a good approximation to the observed dynamical mass.
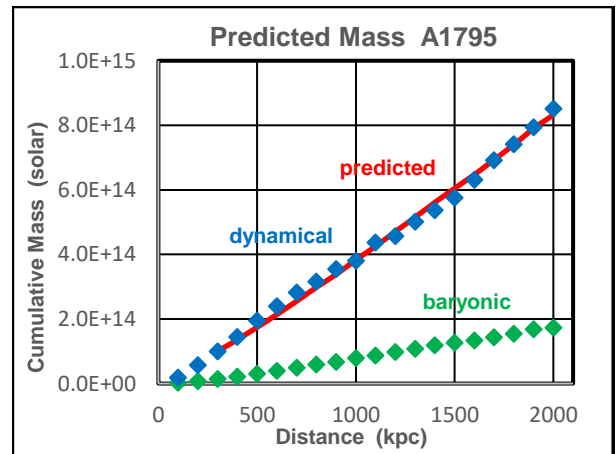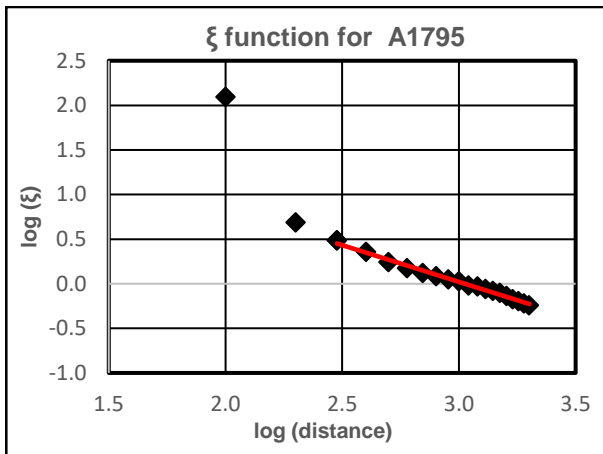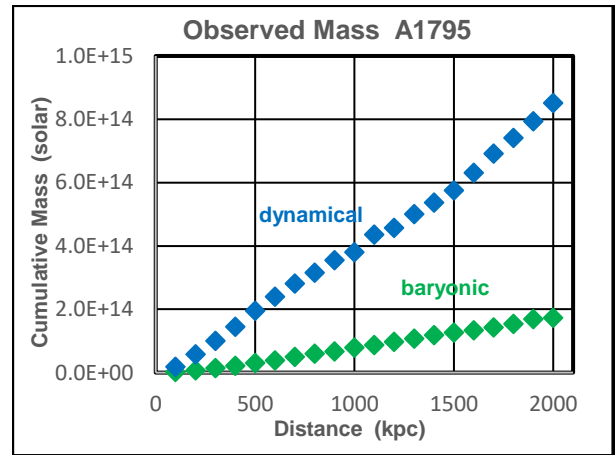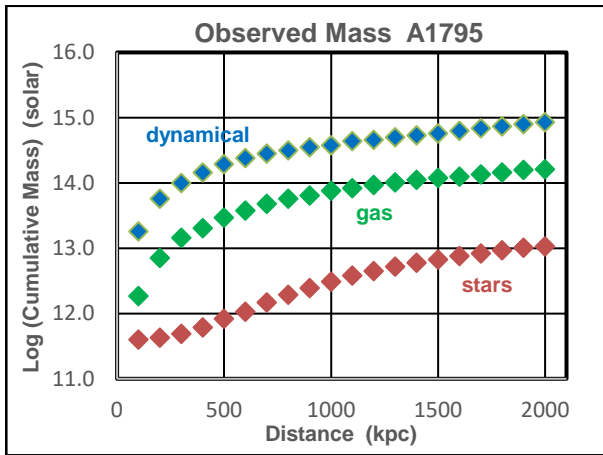
Galaxy Cluster A0085

Slope of ξ-function: -1.05

The red line in the bottom right panel is the predicted curve for the dynamical mass distribution. It is based on the baryonic mass distribution (green diamonds) and a straight line for the ξ-function, similar to that shown in the bottom left panel. The fit is normalised at the 3rd data point.
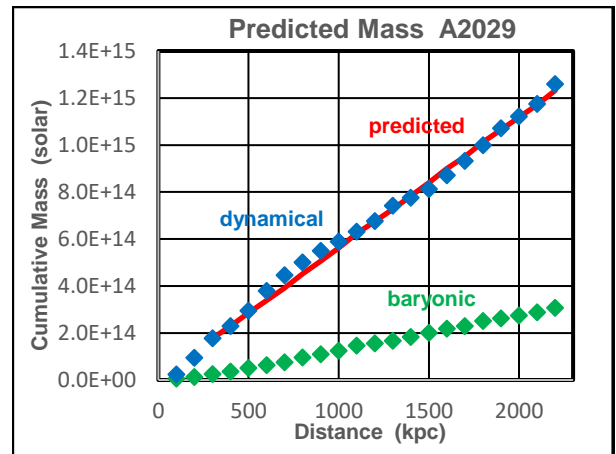
Galaxy Cluster A1795

Slope of ξ-function: -0.89

The red line in the bottom right panel is the predicted curve for the dynamical mass distribution. It is based on the baryonic mass distribution (green diamonds) and a straight line for the ξ-function, similar to that shown in the bottom left panel. The fit is normalised at the 3rd data point.
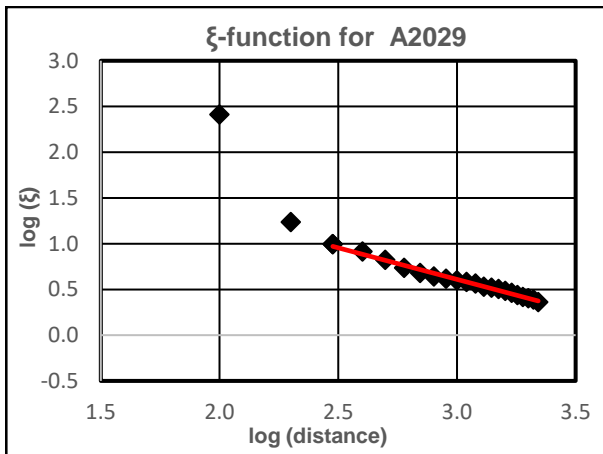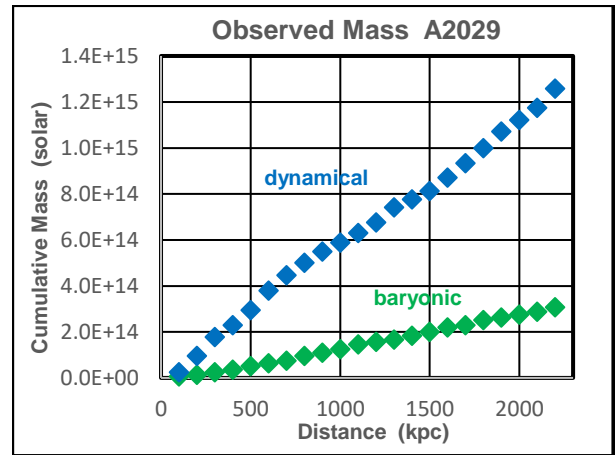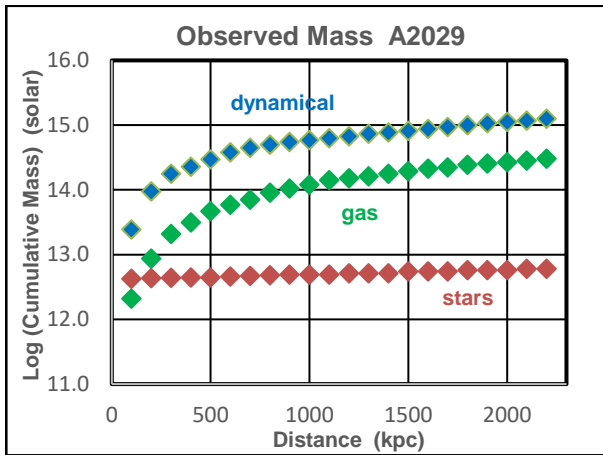
Galaxy Cluster A2029

Observed Mass A2029 (top left panel — Log Cumulative Mass vs Distance, showing dynamical, gas, stars)

Observed Mass A2029 (top right panel — Cumulative Mass vs Distance, showing dynamical, baryonic)

ξ-function for A2029 (bottom left panel — log(ξ) vs log(distance))

Predicted Mass A2029 (bottom right panel — Cumulative Mass vs Distance, showing predicted, dynamical, baryonic)

Slope of ξ-function:　　　-0.73

The red line in the bottom right panel is the predicted curve for the dynamical mass distribution. It is based on the baryonic mass distribution (green diamonds) and a straight line for the ξ-function, similar to that shown in the bottom left panel. The fit is normalised at the 3rd data point.
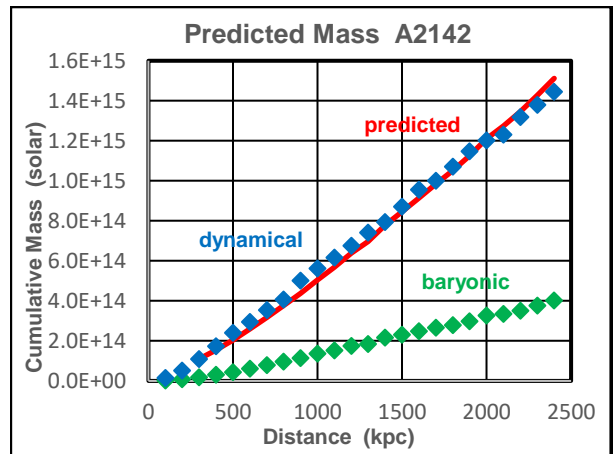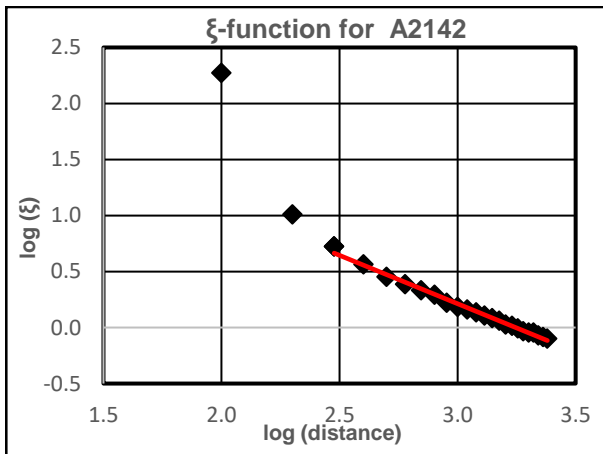
Galaxy Cluster A2142

Slope of ξ-function: -0.93

The red line in the bottom right panel is the predicted curve for the dynamical mass distribution. It is based on the baryonic mass distribution (green diamonds) and a straight line for the ξ-function, similar to that shown in the bottom left panel. The fit is normalised at the 3rd data point.
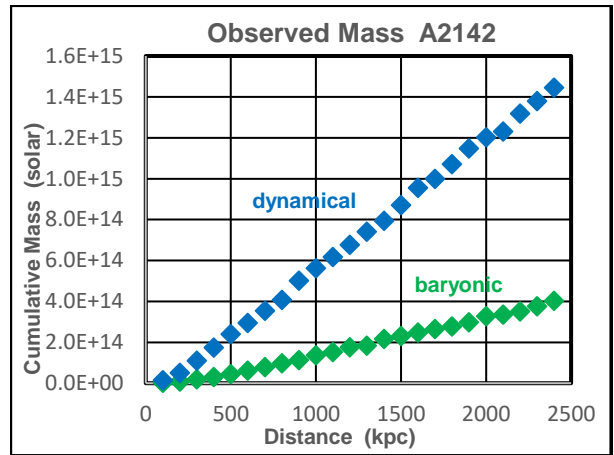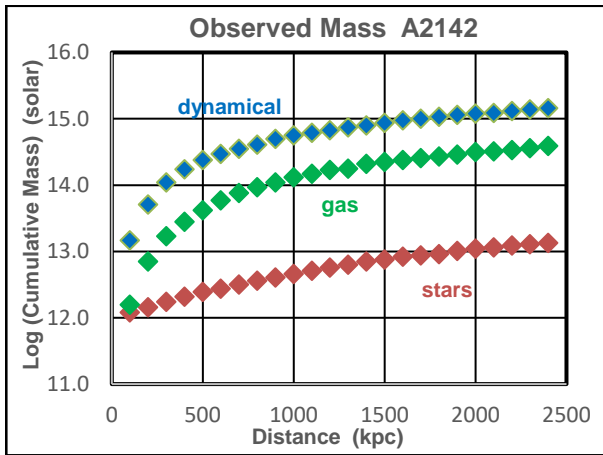
Galaxy Cluster A3158

Slope of ξ-function: -1.20

The red line in the bottom right panel is the predicted curve for the dynamical mass distribution. It is based on the baryonic mass distribution (green diamonds) and a straight line for the ξ-function, similar to that shown in the bottom left panel. The fit is normalised at the 3rd data point.
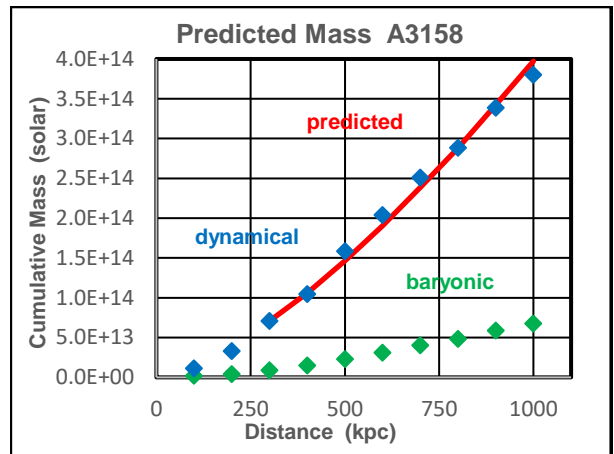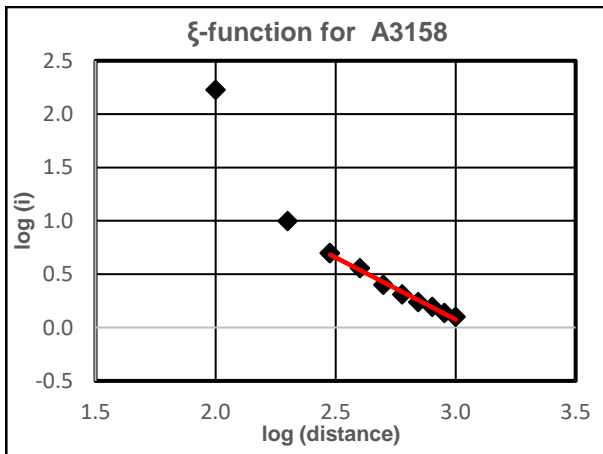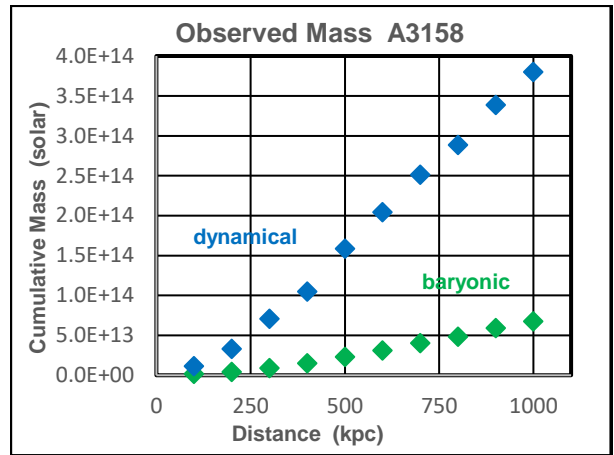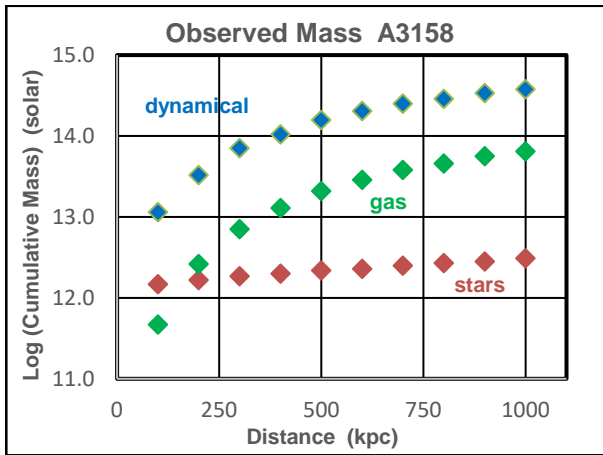
5   A linear relationship

5.1   The bottom left panels for the disk galaxies and galaxy clusters all show a strong linear relationship between the logarithms of the ξ-function and distance. This is an observational result based on the measured baryonic masses and the assumption that the energy scale varies from location to location. It was entirely unexpected. The connection between the baryonic and dynamical masses, as given by equation (3) for disk galaxies and (4) for galaxy clusters, gave no hint that such a linear relationship for the ξ-function might exist.

5.2   The observed linear relationship means that the ξ-function is given by

$$\log(\xi) \; = \; \alpha \, \log(r) \; + \; \text{constant} \tag{5}$$

where α is the slope of the linear relationship.
The equation can be rewritten, without the logarithms, as

$$\frac{\xi(r)}{\xi_0} \; = \; \left(\frac{r}{r_0}\right)^{\alpha} \tag{6}$$

where the equation is normalised at the point (ξ_0, r_o), The observations show that the exponent α (the slope of the linear relationship) lies in the range

$$-0.5 \; > \; \alpha \; > \; -1.8 \tag{7}$$

5.3   It is encouraging that the linear relationship, originally found for disk galaxies (JoKe, 2019), applies equally well to galaxy clusters. Of course, any hypothesis for explaining the mass discrepancies found in many astronomical scenarios, must be able to explain multiple scenarios, not just one. Nevertheless, it is a big step forward for the conjecture, that the energy scale can vary from location to location, that it can explain both disk galaxies and galaxy clusters.

5.4   It is not surprising that the exponent, α, is not a fixed constant, but varies from galaxy to galaxy and from cluster to cluster. Just as disk galaxies and galaxy clusters come in different sizes and masses, so we would expect our energy scale variations to come in different shapes and sizes. However, it is surprising that the α exponent is the only parameter that is required. We can now explain the dynamical masses of both disk galaxies and galaxies clusters using an equation with just one free parameter, the α exponent; a different value of the α exponent is needed for each object.
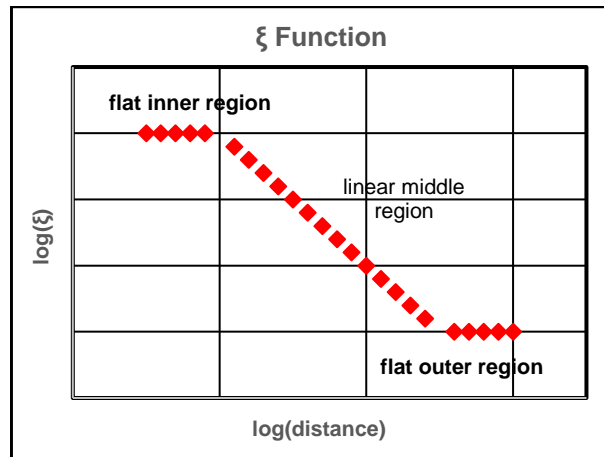
Figure 4. Expected behaviour of ξ-function. The logarithm of the ξ-function cannot be linear over its entire range. It must level off in the innermost regions and in the outermost regions where it merges into intergalactic space.

5.5   Although the data for both disk galaxies and galaxy clusters show a strong linear relationship, the logarithm of the ξ-function cannot stay linear over its entire range. Our expected behaviour is illustrated in Figure 4. The log function has an obvious discontinuity in the central regions as the distance shrinks to zero and the log function goes to infinity. Therefore we suggest the ξ-function must level off to a fixed value at the centre. At large distances both galaxies and clusters end and we enter into intergalactic space where, again, we expect the ξ-function to level off to another fixed value.

6   Discussion

6.1   We now have three ways of explaining the mass discrepancy or dark matter problem. We can illustrate this through gravitational acceleration, g.

(a)   Dark Matter. The mass discrepancy problem is solved by the addition of large amounts of dark matter, which remains hypothetical and has never been detected in any experiment. The equation of gravitational acceleration for mass M at distance r is given by

$$g = -\frac{G\ (M + M_{DM})}{r^2} \tag{8}$$

where M_{DM} is the dark matter that is added to give the required acceleration.

(b)   Modified Gravity. The law of gravity is changed. The best example is MOND proposed by Milgrom (Sanders, 2010), where the acceleration in regions of very low acceleration is given by

$$g = -\sqrt{\frac{G\ M}{r^2}\ a_0} \tag{9}$$

where a_0 is the limiting acceleration (~$1.2{\times}10^{-10}$ m.s^{-2}).

(c)   Variation of the energy scale. For our conjecture the acceleration is given by

$$g = -\frac{G}{r^2}\ \left(\frac{\xi_0}{\xi_r}\right) M \tag{10}$$

where the baryonic mass is multiplied by the ration of ξ values to give the dynamical mass.

6.2   The conjecture, that the energy scale varies from location to location, works for both disk galaxies and galaxy clusters. This has been demonstrated in the previous sections and they clearly provide a strong supporting case. This linear relationship is essentially an observational result. It can be tested on any other astronomical object, where baryonic and dynamical measurements are available; an example would be globular clusters.

6.3   The data used for analysing the galaxy clusters was obtained by measuring the logarithmic plots published results of Li et al (2023). Measuring logarithmic data and then converting it to linear values is never a good idea and can introduce large errors. We should repeat the analysis as soon as the linear data values become available. Nevertheless, we believe the linear relationship we obtained for galaxy clusters is completely correct and valid.

6.4   The conjecture can only be tested by having objects in different locations. This rules out any experiments carried out on Earth. For disk galaxies and galaxy clusters the ξ-function varies very slowly with distance. This probably rules out any experiments carried out in the solar system where we estimate the change to be below 1 part in 10^{10}.

6.5   Of course, dark matter is used to explain a wider range of astronomical scenarios, not just disk galaxies and galaxy clusters. Dark matter is involved in the cosmic microwave background and the formation of structure. It is not clear, yet, how energy scale variations might help there. However, gravitational lensing is not a problem as that depends on the dynamical mass for which the conjecture supplies a complete answer.

6.6   Our conjecture involves the introduction of the ξ-function to describe the variation of the energy scale. This is a scalar field of position and has a different value at each location. As a scalar field it should share properties with other scalar fields. A separate paper "Towards a theory of energy scale variations" is available at www.varensca.com. The paper goes deeper into considerations of the physics and mathematics; these have only been dealt with in a cursory fashion here. The paper considers several topics including how the conjecture affects potential theory and items such as Gauss' Law and Poisson's Equation.

6.7   At the moment the idea that the energy scale can vary is just a conjecture. There is no concrete evidence that they exist. The paper "Predictions and Tests" (Jo.Ke, 2019) put forward a number of predictions and tests, many of which can be carried out now; these enable the conjecture to be tested and falsified. There is no proper theory for energy scale variations, and we have no idea how to create an energy scale variation, should they exist. These are all matters for future work.

7   References

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