On the variation of the energy scale 29

Towards a theory of energy scale variations: an alternative to dark matter

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Summary

We have put forward the conjecture that the energy scale can vary from location to location, and showed that this simple idea can explain all those astronomical situations where dark matter is assumed to exist. So, without invoking dark matter, our conjecture explains: the rotation curves of spiral galaxies; the velocities of galaxies in clusters; gravitational lensing; the acoustic peaks in the power spectrum of the cosmic microwave background; structure formation; and much more. In addition our conjecture leads to a number of predictions that can be tested. However, as yet we do not have a proper theory for variations of the energy scale. In this paper we present our latest work towards establishing a firm basis for such a theory.

1x. Introduction

1x.1 All physical quantities (such as length, speed, pressure) can be expressed in terms of a number of different scales or dimensions. Dimensional analysis shows that only a small number of scales are needed to describe physical quantities with the main ones used by the International System of Units (SI) being: mass; length; time; electric current. For our work here and in previous papers, we switch from a mass scale to an energy scale and we also assume that none of the other scales can vary. So, it is only the energy scale that can vary from place to place.

1x.2 In the paper "On the variation of the energy scale: an alternative to dark matter" (JoKe1, 2015) we introduced the idea of variations of the energy scale to explain the rotation curves of spiral galaxies. Additional papers extended this to cover all astronomical scenarios where dark matter is invoked, including clusters of galaxies, gravitational lensing, and the cosmic microwave background. These showed that we do not need dark matter to explain any of the scenarios, variations of the energy scale provide an alternative explanation to all of them. We summarised all our work in JoKe27 (2020).

1x.3 It turns out that the astronomical scenarios, where dark matter is invoked, are either static, unchanging, or in equilibrium. For galaxy rotation curves circular orbits inside a fixed dark matter halo are assumed; so, nothing is changing. For gravitational lensing it is again assumed that a fixed dark matter halo exists. Although galaxies are not static the distances and time-scales involved make it impossible to observe any changes. This makes it extremely difficult to understand how dark matter behaves and to get at its properties.

1x.4 One of the best scenarios for watching how dark matter behaves should be galaxy interactions. If we could follow the collision of two galaxies, then we would have a complete understanding of how the baryonic matter and the dark matter interact with one another. Unfortunately, all we have is a single snapshot of such collisions a single moment in time. However, we may be able to use computer simulations to improve our theory or to eliminate some ideas.

2x. An example: galaxy rotation curves

2x.1 Before getting too involved in the theory of variations of the energy scale, we need to explain what we are talking about and this is best done through the example of the rotation curves of spiral galaxies. The shape of the rotation curve in the outer part of spiral galaxies is observed to be roughly flat, which is in sharp disagreement with the expected fall off for Newtonian gravity.

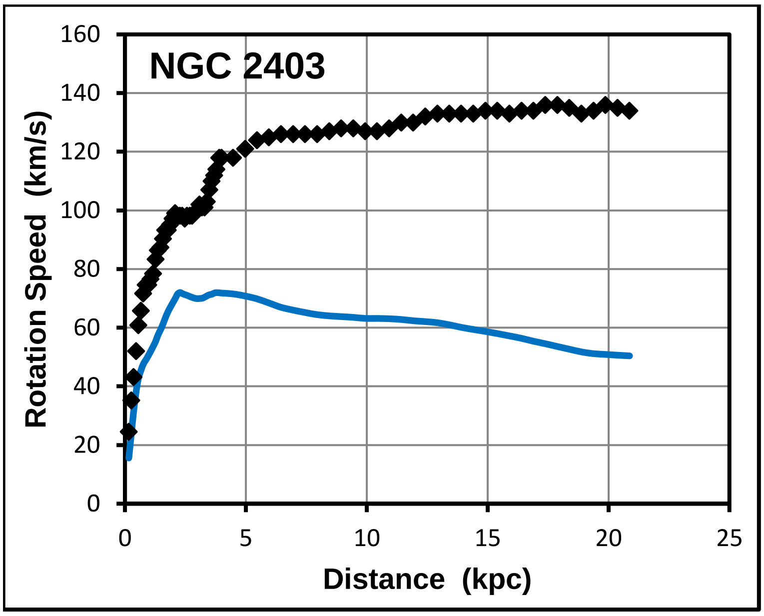

Fig 1. The rotation curve for spiral galaxy NGC 2403, based on data from the SPARC catalog. The black diamonds are the observations; the solid blue curve is the expected curve for Newtonian gravitation.

2x.2 Fig 1 shows the rotation curve for spiral galaxy NGC 2403, using data from the SPARC catalogue (Lelli et al, 2016). The black diamonds are the observations; the solid blue line is the expected rotation curve based on the observed mass distribution. The discrepancy between the observed and expected curves is clear; the usual explanation for this discrepancy is that the galaxy is embedded in a large halo of dark matter.

2x.3 We work with Newton's law of gravity and that the rotational velocity is given by

$$\frac{v^2}{r} = \frac{G\,M(r)}{r^2} \tag{1}$$

where $M(r)$ is the mass interior to radius r. (This is really only correct for systems that are spherically-symmetric but, as discussed later, this is a good approximation for spiral galaxies.) The left hand side of equation (1) is the centrifugal acceleration; the right hand side is the gravitational acceleration. For circular orbits the two sides balance.

2x.4 We can use equation (1) to predict the rotational velocity, v , from the observed baryonic mass distribution, $M_B(r)$

$$v^2 = \frac{G\, M_B(r)}{r} \tag{2}$$

When we do this for NGC 2403 we get the blue line in Fig 1. Clearly there is a problem in that the observed baryonic mass distribution and Newtonian gravity in the form of equation (2) do not explain the observed rotational velocity curve.

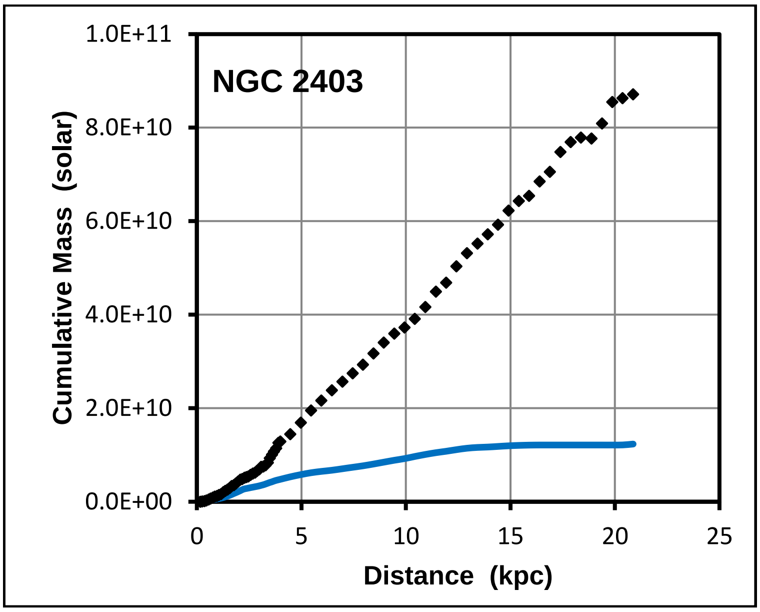

Fig 2. Cumulative mass (black diamonds) required to explain the observed rotation curve of spiral galaxy NGC 2403. The solid blue line is the observed mass distribution.

2x.5 The usual way to explain Fig 1 is to postulate the existence of a large spherical halo of (non-baryonic) dark matter. Equation (8) then becomes

$$v^2 = \frac{G\,\{M_B(r) + M_D(r)\}}{r} \tag{3}$$

where $M_D(r)$ is the mass of non-baryonic dark matter interior to radius r. We cannot predict the amount of dark matter in advance. Instead we simply invert equation (3) and use it to tell us how much dark matter is required to explain the observations

$$M_D(r) = \frac{r\,v^2}{G} - M_B(r) \tag{4}$$

This is illustrated in Fig 2 (above). The solid blue line shows $M_B(r)$, the observed amount of baryonic mass inside radius r. The black diamonds show $M_B(r)+M_D(r)$, the total mass inside radius r required to explain the observed rotation curve. Roughly five times as much dark matter than baryonic matter is needed. Although the baryonic mass converges around 12 kpc, the dark matter mass is still increasing even at the outer limits of the galaxy.

2x.6 Our conjecture of variations of the energy scale uses a totally different approach to explaining galaxy rotation curves. If the energy scale varies across a galaxy, then (as explained fully later on) equation (8) is replaced by

$$v^2 = \frac{G}{r}\,\frac{\xi_A\;M_B(r)}{\xi(r)} \tag{5}$$

where ξ is the scalar function that describes the energy scale variation. ξ_A is the value interior to radius r; $\xi(r)$ is the value at radius r. So the gravitational effect by a remote mass on an object depends on the ratio of the ξ function at the remote mass and the object.

Whereas the dark matter hypothesis introduces an additive factor (the dark matter halo) to explain galaxy rotation curves, our hypothesis of energy scale variations introduces a multiplicative factor (the ξ function).

It is important to note that our ξ function acts on the mass and changes the effective mass of the attracting body. It does not change the nature of Newtonian gravitational force, which still depends on the product of the masses and inversely on the square of the distance.

2x.7 We can get at the values of the ξ function by inverting equation (5) as

$$\frac{\xi(r)}{\xi_A} = \frac{G\;M_B(r)}{r\;v^2} \tag{6}$$

This is shown in Fig 3 for spiral galaxy NGC 2403. The black diamonds are the values corresponding to the observations. The striking feature of this plot is the almost perfect straight line formed by the data points. This feature was unexpected and came as a complete surprise. The solid red line is a straight line approximation to the black diamonds. The straight line is not special to NGC 2403; all spiral galaxies show a similar linear relationship.

2x.8 Having established the linear relationship for our ξ function, we can use this to construct our own rotation curve by using equation (5). The result of this is shown in Fig 4. This is the same as Fig 1, with the addition of the solid red line through the observed data points, which is our rotation curve. The fit is clearly very good and gives us considerable confidence that our conjecture of variations of the energy scale is worthy of further examination.

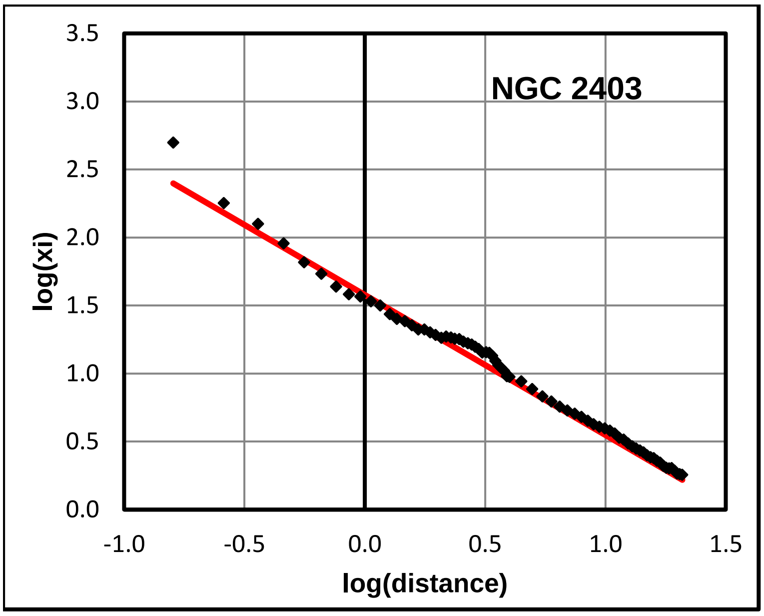

Fig 3. Graph of the energy scale variation function, ξ, for spiral galaxy NGC 2403. function. The black diamonds are derived from equation (6) and they show a clear linear relationship; the solid red line is a linear fit. All spiral galaxies show a similar linear relation.

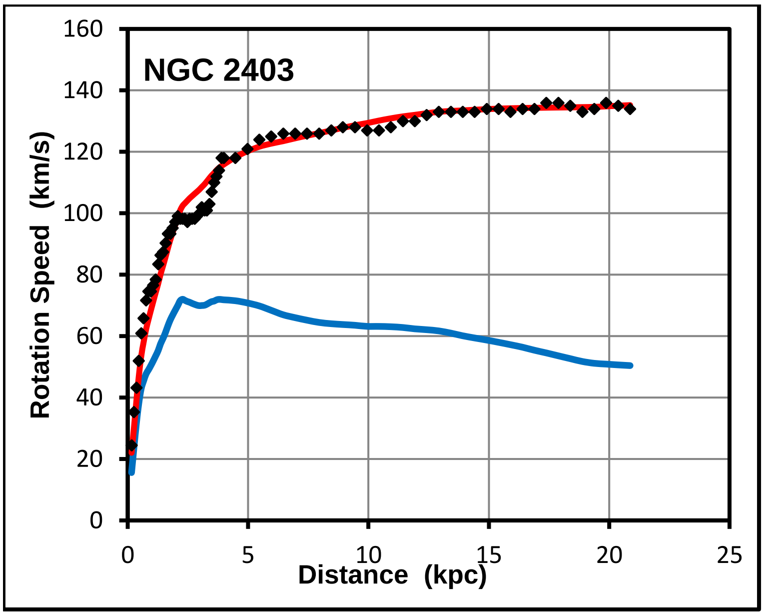

Fig 4. The rotation curve for spiral galaxy NGC 2403. The black diamonds are the observations; the solid blue line is the expected curve assuming Newtonian gravity;

the solid red line is the curve generated by assuming the straight line approximation for the ξ function as shown in Fig 3; i.e. using equation (5).

2x.9 For spiral galaxies the ξ function, describing the variation of the energy scale, has higher values near the galaxy centre and lower values outwards through the spiral arms. For stars in the spiral arms the interior mass behaves as the actual (intrinsic) mass multiplied by the high value for the interior ξ function and divided by the low value of the local ξ function. These values lead to the central mass behaving with a greatly enhanced mass, which in turn leads the high rotational velocity and flat rotation curve.

2x.10 The observed linear relationship of the ξ function means we can predict the shape of the rotation curve of spiral galaxies. We can use the observations of the inner part of the rotation curve and equation (6) to establish the slope of the straight line approximation to the ξ function. We can then predict the rest of the rotation curve using equation (5). The solid red line in Fig 4 shows the result of this procedure for spiral galaxy NGC 2403. Although not a perfect fit, the conjecture that the energy scale can vary results in a very good fit to the observed rotation curves of spiral galaxies. We note that the dark matter hypothesis cannot predict the shape of rotation curves to the same extent.

2x.11 This section has given a brief introduction to variations in the energy scale and shown how, by allowing the energy scale to vary from location to location, we can explain the observed rotation curves of spiral galaxies. The ξ function that defines energy scale variations is a scalar function of position, i.e. a scalar field. There is no direct evidence that such a scalar field exists nor that variations in the energy scale do occur. But, similarly with the hypothesis of dark matter, there have been no direct detections of any dark matter particles.

Now we need to go back, look at our conjecture of energy scale variations, see how it fits in with existing physical theories, and see whether we can put it on a firm theoretical basis. These tasks are what we will be considering over the next sections.

3x. The conjecture

3x.1 Our conjecture for variations of the energy scale can be stated very simply as:

The energy scale can vary from location to location

3x.2 We need to define what we mean by "energy scale". Standard dimensional analysis expresses physical quantities in terms of a number of base quantities. The International System of Units (SI) works with seven base quantities or dimensions; chief amongst these are length, time, mass, and electric current. Instead of mass, we are going to work with energy as it covers a broader range of concepts than just mass. Dimensional analysis uses a number of different words, including: dimension, unit, quantity and scale. We are going to stick with the word "scale". So we are going to be talking about variations of "the energy scale" and not "the energy dimension" or "the energy unit".

3x.3 We assume that it is only the energy scale that varies. All the other scales (length, time, electric current, etc) are assumed to be fixed which, amongst other things, means the speed of light is an absolute constant. So no changes in this respect for Special Relativity.

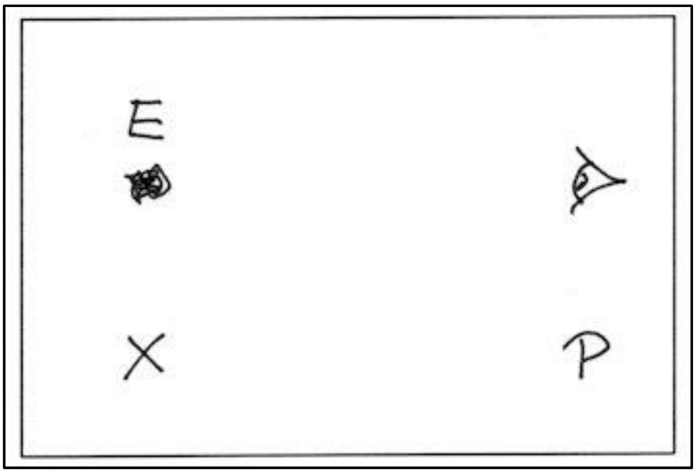

Fig 3y_1. Situation for an energy E at location X, with no variation of the energy scale.

3x.4 We start with an energy E at location X , an observer at location P , and no variation of the energy scale. This situation is illustrated in Fig 3y_1. Both an observer with the energy at X and an observer at P agree that the energy has a value E .

3x.6 We need to introduce a notation that enables us to work with quantities, objects and observers that are placed at different locations. We use the notation

$$\boldsymbol{E_X^P} \equiv \boldsymbol{E}_{\textbf{Quantity At}}^{\textbf{By Observer}} \tag{7}$$

where a subscript denotes the location of the quantity (object), and a superscript denotes the location of the observer. So $\boldsymbol{E_X^P}$ is the value of the energy at location $\boldsymbol{X}$ as measured by an observer at location $\boldsymbol{P}$.

3x.7 For our conjecture we also need to introduce a scalar field, ξ, that defines the strength of the energy scale at each location.

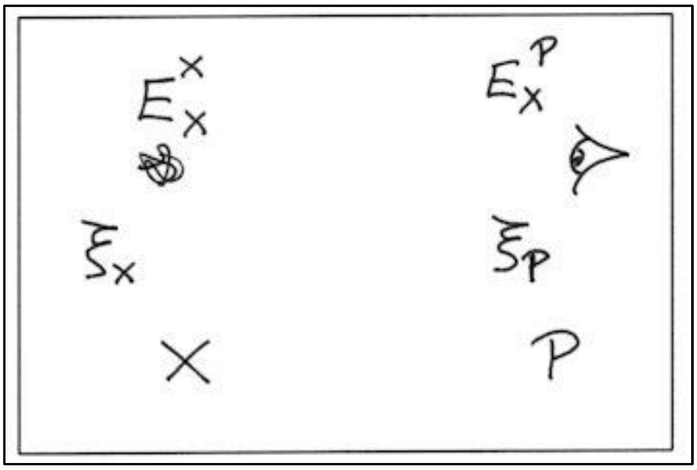

Fig 3y_2. Situation for an energy $\boldsymbol{E}$ at location $\boldsymbol{X}$ and a remote observer at $\boldsymbol{P}$, with a variation of the energy scale, ξ.

3x.8 We consider an energy $\boldsymbol{E}$ at location $\boldsymbol{X}$ and an observer at location $\boldsymbol{P}$, as illustrated in Fig 3y_2. It follows that our conjecture can be expressed as

$$\xi_P \, \boldsymbol{E_X^P} = \xi_X \, \boldsymbol{E_X^X} = \xi_X \, \boldsymbol{E} \tag{8}$$

where ξ is the dimensionless function of position that describes how the energy scale varies.

ξ_X is the value of the ξ function at location $\boldsymbol{X}$.

$\boldsymbol{E_X^X}$ is the energy at $\boldsymbol{X}$ as measured by an observer $\boldsymbol{X}$.

ξ_P is the value of the ξ function at location $\boldsymbol{P}$.

$\boldsymbol{E_X^P}$ is the energy at $\boldsymbol{X}$ as measured by an observer at $\boldsymbol{P}$.

3x.9 When the subscript and superscript are the same, the energy is interpreted as the "intrinsic energy"; in this case it is the energy at $\boldsymbol{X}$ as measured by an observer at $\boldsymbol{X}$, i.e. both object and observer are at the same location. Generally, we can drop the subscripts and superscripts for intrinsic values. However, for clarity in some situations, we may retain both subscripts and superscripts.

We note in equations like equation (8) that the subscript on the ξ function agrees with the superscript on the quantity. This is because we are relating a quantity at a given location with the measurements of observers at different locations. This is more apparent if we extend equation (8) to include location $\boldsymbol{Q}$

$$\xi_X\, E_X^X \;=\; \xi_P\, E_X^P \;=\; \xi_Q\, E_X^Q \tag{9}$$

The subscript on the energy, $\boldsymbol{E}$, is the same throughout as we are talking about the energy at a particular location; whereas the subscript on ξ agrees with the superscript on $\boldsymbol{E}$ as we are talking about observers at different locations.

3x.10 As defined above, ξ is a dimensionless scalar field; it is not a pure constant as its value can change from location to location. But, as a scalar, it is Lorentz invariant, which again has implications for Special Relativity. Also, and unlike quantities that contain energy as part of their dimensions (e.g. energy density, pressure, momentum), ξ is not the component of a vector or a tensor.

Also, as ξ is dimensionless, it only has a subscript, denoting its location. So ξ_X is the value of ξ at location $\boldsymbol{X}$. The concept of the value of ξ at a particular location as measured by an observer at another location is not required; its value is the same for all observers at all locations.

3x.11 There is no absolute scale for our ξ function, only relative values. This is clear when we rewrite equation (8) as

$$E_X^P \;=\; \left(\frac{\xi_X}{\xi_P}\right) E_X^X \tag{10}$$

In all equations & expressions involving energy quantities the ξ function always appears as the ratio of pairs of ξ values. In practice this means we are free to normalise the ξ function to an arbitrary value at a particular point; the values at other points then follow. For galaxy NGC 2403 (covered in the previous section) the ξ function was normalised to a value of 1000 near the galaxy centre. Hence the value of around 3.0 in Figure 3.

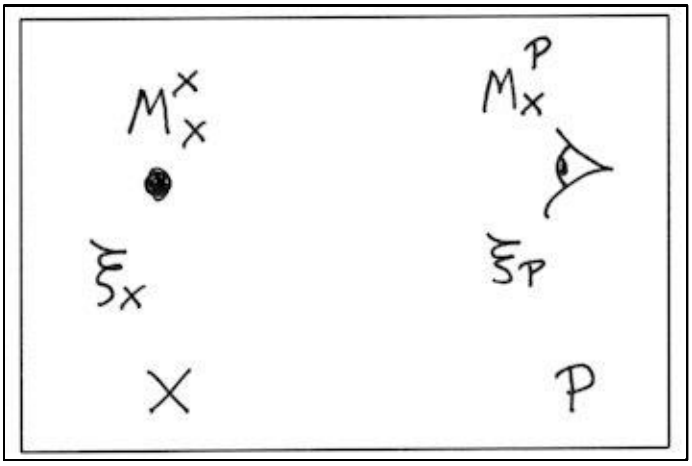

Fig 3y_3. Situation for a mass M at location X and a remote observer at location P, with a variation of the energy scale, ξ.

3x.12 Energy has the dimensions M L^2 T^{-2}, (M:mass, L:length; T:time), and mass is associated with energy through Einstein's equation

$$E = M c^2 \tag{11}$$

where c is the speed of light. As mentioned above, our conjecture means that it is only the energy scale that can change. Hence we can use equation (11) to rewrite equation (8) in terms of mass as

$$\xi_P \left(M_X^P c^2\right) = \xi_X \left(M_X^X c^2\right) \tag{12}$$

or

$$\xi_P M_X^P = \xi_X M_X^X = \xi_X M \tag{13}$$

or

$$M_X^P = M \left(\frac{\xi_X}{\xi_P}\right) \tag{14}$$

where M_X^P is the mass at X as measured by an observer at P. This is illustrated in Figure 3y_3 above. It is equation (14) that we used earlier in our example of galaxy rotation curves; see equation (12) in section 2.

3x.13 As density, ρ, has the dimensions of mass divided by volume, it follows that

$$\xi_P \rho_X^P = \xi_X \rho_X^X = \xi_X \rho \tag{15}$$

where ρ_X^P is the density at X as measured by an observer at P, and ρ_X^X is the density at X as measured by an observer at X.

3x.14 It is not just energies, masses and densities that are affected by our conjecture but all quantities and constants that have energy as part of their units. The energy of a photon involves Planck's constant and is given by

$$\boldsymbol{E} \;=\; \boldsymbol{h}\,\boldsymbol{\nu} \tag{16}$$

Planck's constant has the units of action (energy × time), hence in our new notation

$$\xi_P\, h_X^P \;=\; \xi_X\, h_X^X \;=\; \xi_X\, h \tag{17}$$

where h_X^P is the value of Planck's constant at location X as measured by an observer at P.

Similarly, for the gravitational constant we find

$$\frac{G_X^P}{\xi_P} \;=\; \frac{G_X^X}{\xi_X} \;=\; \frac{G}{\xi_X} \tag{18}$$

where G_X^P is the value of the gravitational constant at location X as measured by an observer at P.

3x.15 Our conjecture means that the values of all quantities that have energy as part of their units will appear to be different to observers in different locations. The ratio of the different values is proportional to the ratio of the energy scales at the different locations. This should be clear from equation (8) for energy and equation (13) for mass. It applies, not just to individual objects like stars or galaxies, but to the constants of physics as well. As in the above examples: Planck's constant appears to have a different value, as does the gravitational constant.

3x.16 All physics equations balance, which means the scales (units) balance as well. For example, if we have length divided by time (speed) on one side of an equation, then we must have length divided by time on the other side. In most physical interactions, all the constituents are located in the same place (e.g. interactions within particle accelerators), which means it is impossible to detect any variations of the energy scale. To detect variations in the energy scale we need the constituents to be placed in different locations. The only situation we are aware of, where possible differences can be measured, is the gravitational interaction between objects at different locations. For example, the gravitational interaction between the Earth and the Sun, or the rotation curve of a spiral galaxy.

3x.17 Our general approach to handling expressions and equations is to replace all physical quantities by their intrinsic values, through the introduction of appropriate values of our ξ function. That is, where we have objects and observers in different locations, we use relations similar to equation (8) to replace every quantity by its value as measured by an observer at the same location. This should become clear in the next section, where we discuss the gravitational interaction of point masses.

4x. Gravity for point masses

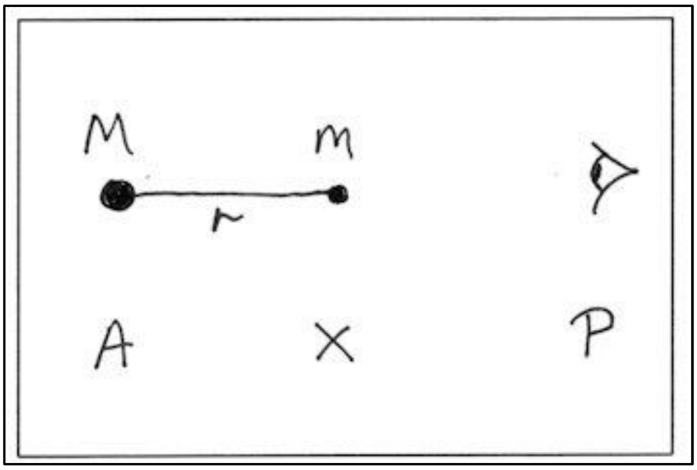

Fig 4y_1. Gravity between two masses, with no variation of the energy scale.

4x.1 Newton's theory of gravitation states that the force $\boldsymbol{F}$ between mass $\boldsymbol{m}$ at location $\boldsymbol{X}$ and mass $\boldsymbol{M}$ at location $\boldsymbol{A}$ a distance $\boldsymbol{r}$ away is

$$\boldsymbol{F} \;=\; \boldsymbol{m}\,\ddot{\boldsymbol{r}} \;=\; -\,\frac{\boldsymbol{G\,m\,M}}{\boldsymbol{r^2}} \tag{19}$$

This is illustrated in Fig 4y_1. The force is symmetrical in that the magnitude of the force on both masses is the same. It follows that the acceleration on mass $\boldsymbol{m}$ is given by

$$\ddot{\boldsymbol{r}} \;=\; -\,\frac{\boldsymbol{G\,M}}{\boldsymbol{r^2}} \tag{20}$$

4x.2 We now look at how this equation changes for our conjecture of variations of the energy scale. We remember that a subscript denotes the location of the quantity, and a superscriptcript denotes the location of the observer. We use the same situation as above, but the labelling changes as illustrated in Fig 4y_2. The acceleration experienced by mass $\boldsymbol{m}$ at location $\boldsymbol{X}$, as measured by an observer also at $\boldsymbol{X}$, is given by

$$\ddot{\boldsymbol{r}}_X^X \;=\; -\,\frac{\boldsymbol{G_X^X\,M_A^X}}{\boldsymbol{r^2}} = -\,\frac{\boldsymbol{G}}{\boldsymbol{r^2}}\left(\boldsymbol{M_A^A}\,\frac{\boldsymbol{\xi_A}}{\boldsymbol{\xi_X}}\right) = -\,\frac{\boldsymbol{G\,M}}{\boldsymbol{r^2}}\left(\frac{\boldsymbol{\xi_A}}{\boldsymbol{\xi_X}}\right) \tag{21}$$

where, as defined above, $\boldsymbol{\xi}$ is the dimensionless function of location that describes the variation of the energy scale, and we have used equation (14) to transform the mass.

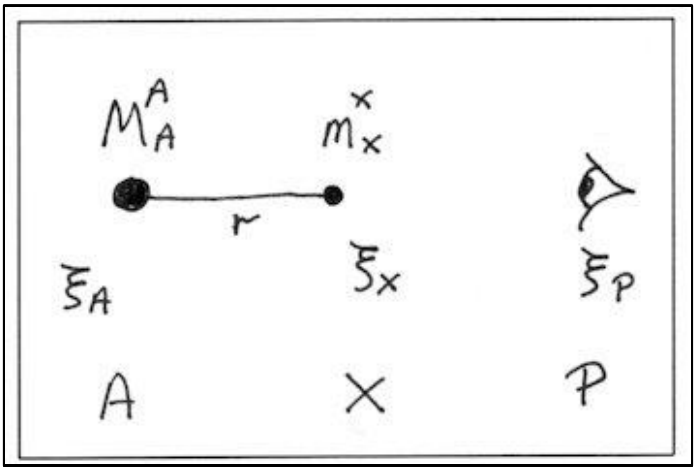

Fig 4y_2. Gravity between two masses with a variation of the energy scale.

The same situation as measured by an observer at $\boldsymbol{A}$, is given by

$$\ddot{\boldsymbol{r}}_X^A = -\frac{G_X^A\,M_A^A}{r^2} = -\frac{M_A^A}{r^2}\left(G_X^X\,\frac{\xi_A}{\xi_X}\right) = -\frac{G\,M}{r^2}\left(\frac{\xi_A}{\xi_X}\right) \qquad (22)$$

where we have used equation (18) to transform $\boldsymbol{G}$.
And the same situation as measured by a remote observer at $\boldsymbol{P}$, is given by

$$\ddot{\boldsymbol{r}}_X^P = -\frac{G_X^P\,M_A^P}{r^2} = -\frac{1}{r^2}\left(G_X^X\,\frac{\xi_P}{\xi_X}\right)\left(M_A^A\,\frac{\xi_A}{\xi_P}\right) = -\frac{G\,M}{r^2}\left(\frac{\xi_A}{\xi_X}\right) \qquad (23)$$

where we have used both equations (14) and (18).
So, we end up with the same result, as we must, irrespective of whether we consider location $\boldsymbol{X}$ or location $\boldsymbol{A}$, or remote location P.
We should also note that the units of acceleration are length divided by time squared; the energy scale is not involved. So, naturally, the acceleration measured by all observers has to be the same.

4x.3 The above section (4x.2) shows our general technique for handling equations where the quantities and observers are at different locations. We use expressions, similar to equation (8), to replace every quantity with its intrinsic value, i.e. the value of the quantity as measured by an observer at the same location. This process introduces various values of our dimensionless function of position, $\boldsymbol{\xi}$.

4x.4 In potential theory the force is the gradient of the scalar potential and conversely the potential energy is the integral of the force. So, in principle we can get at the gravitational potential by integrating equation (19), which for our conjecture is replaced by equation (21). However, $\boldsymbol{\xi}$ is a function of position, which makes it difficult to simply integrate equation (21) to get at the gravitational potential. We can make this clearer by writing equation (21) as

$$\ddot{\boldsymbol{r}} \;=\; -\,\boldsymbol{G\,M}\,\xi_A \left(\frac{\mathbf{1}}{\boldsymbol{r}^2\ \ \xi_X(\boldsymbol{r})} \right) \tag{24}$$

We do not yet have a theory for energy scale variations. We do not know the functional form of our $\xi_X(\boldsymbol{r})$ function and so cannot integrate this equation. This is discussed further in the next section on potential theory.

4x.5 It is instructive to look at the gravitational force. We stick with our same arrangement of masses and locations. The force on mass $\boldsymbol{m}$ at $\boldsymbol{X}$ due to mass $\boldsymbol{M}$ at $\boldsymbol{A}$ as measured by an observer at $\boldsymbol{P}$ is, by extending equation (23)

$$\boldsymbol{F}_X^P \;=\; \boldsymbol{m}_X^P\ \ddot{\boldsymbol{r}}_X^P \;=\; \boldsymbol{m}_X^X \left(\frac{\xi_X}{\xi_P}\right) \ddot{\boldsymbol{r}}_X^P \;=\; -\,\frac{\boldsymbol{G\,M\,m}}{\boldsymbol{r}^2} \left(\frac{\xi_A}{\xi_P}\right) \tag{25}$$

It is interesting that, although we are talking about the force at $\boldsymbol{X}$, this equation does not involve ξ_X , the value of the energy scale at $\boldsymbol{X}$.

4x.6 The following table gives the force for the full set of locations and observers, i.e. mass $\boldsymbol{m}$ at $\boldsymbol{X}$; mass $\boldsymbol{M}$ at $\boldsymbol{A}$; remote observer at $\boldsymbol{P}$. The entries follow from our relation for force

$$\xi_X\ \boldsymbol{F}_X^X \;=\; \xi_A\ \boldsymbol{F}_X^A \;=\; \xi_P\ \boldsymbol{F}_X^P \tag{26}$$

and

$$\xi_X\ \boldsymbol{F}_A^X \;=\; \xi_A\ \boldsymbol{F}_A^A \;=\; \xi_P\ \boldsymbol{F}_A^P \tag{27}$$

where we remember that a superscript denotes the location of the quantity, and a subscript the location of the observer.

Force	Equation	Description
$\boldsymbol{F}_X^X$	$-\,\frac{\boldsymbol{G\,M\,m}}{\boldsymbol{r}^2}\left(\frac{\xi_A}{\xi_X}\right)$	Force on $\boldsymbol{m}$ at $\boldsymbol{X}$ due to $\boldsymbol{M}$ at $\boldsymbol{A}$, as measured by $\boldsymbol{X}$
$\boldsymbol{F}_X^A$	$-\,\frac{\boldsymbol{G\,M\,m}}{\boldsymbol{r}^2}$	Force on $\boldsymbol{m}$ at $\boldsymbol{X}$ due to $\boldsymbol{M}$ at $\boldsymbol{A}$, as measured by $\boldsymbol{A}$
$\boldsymbol{F}_X^P$	$-\,\frac{\boldsymbol{G\,M\,m}}{\boldsymbol{r}^2}\left(\frac{\xi_A}{\xi_P}\right)$	Force on $\boldsymbol{m}$ at $\boldsymbol{X}$ due to $\boldsymbol{M}$ at $\boldsymbol{A}$, as measured by $\boldsymbol{P}$
$\boldsymbol{F}_A^X$	$+\,\frac{\boldsymbol{G\,M\,m}}{\boldsymbol{r}^2}$	Force on $\boldsymbol{M}$ at $\boldsymbol{A}$ due to $\boldsymbol{m}$ at $\boldsymbol{X}$, as measured by $\boldsymbol{X}$
$\boldsymbol{F}_A^A$	$+\,\frac{\boldsymbol{G\,M\,m}}{\boldsymbol{r}^2}\left(\frac{\xi_X}{\xi_A}\right)$	Force on $\boldsymbol{M}$ at $\boldsymbol{A}$ due to $\boldsymbol{m}$ at $\boldsymbol{X}$, as measured by $\boldsymbol{A}$

Force	Equation	Description
$\boldsymbol{F_A^P}$	$+\dfrac{\boldsymbol{G\,M\,m}}{\boldsymbol{r^2}}\left(\dfrac{\boldsymbol{\xi_X}}{\boldsymbol{\xi_P}}\right)$	Force on $\boldsymbol{M}$ at $\boldsymbol{A}$ due to $\boldsymbol{m}$ at $\boldsymbol{X}$, as measured by $\boldsymbol{P}$

4x.7 Gravity and Newton's 3rd Law

It is clear from the above table that, for the remote observer P, Newton's 3rd Law (action and reaction are equal and opposite) is broken. The force at $\boldsymbol{X}$ is not equal and opposite to the force at $\boldsymbol{A}$

$$\boldsymbol{F_X^P} \;\neq\; -\boldsymbol{F_A^P} \tag{28}$$

Instead, the table also shows that

$$\boldsymbol{F_X^A} \;=\; -\boldsymbol{F_A^X} \tag{29}$$

i.e. the force on $\boldsymbol{m}$ at $\boldsymbol{X}$ as measured by $\boldsymbol{A}$ is equal and opposite to the force on $\boldsymbol{M}$ at $\boldsymbol{A}$ as measured by $\boldsymbol{X}$.

4x.8 In summary, we still have Newton's theory of gravity. It still depends on the product of the masses and on the inverse square of the distance. What we have though is the additional complication of our $\boldsymbol{\xi}$ function. What we now require is a theory that defines the nature of the $\boldsymbol{\xi}$ function and the variations of the energy scale.

5x. Another look at mass

5x.1 In the previous section we looked at point masses. However, most masses occur not as points but as extended bodies with different shapes and with varying densities. We now consider how our conjecture changes the way we have to work with such bodies. We also work with a simple physical situation, with no relativistic complications.

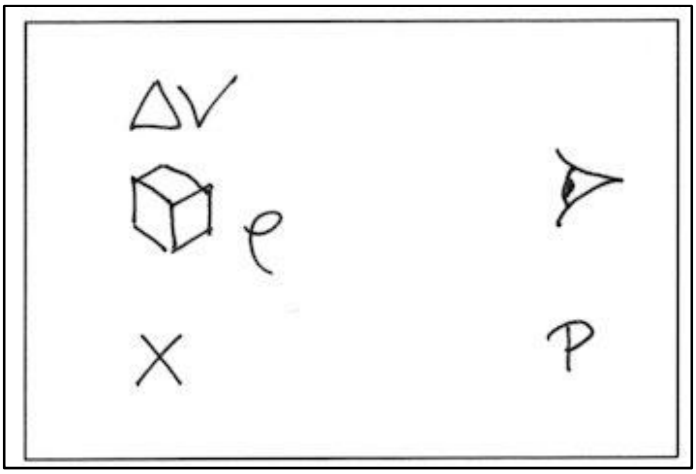

Fig 5y_1. Mass increment at location $\boldsymbol{X}$ and a remote observer at location $\boldsymbol{P}$, with no variation of the energy scale.

5x.2 We consider a small increment of mass, $\boldsymbol{\Delta M}$ at location $\boldsymbol{X}$; volume element $\boldsymbol{\Delta V}$ and density $\boldsymbol{\rho}$, as illustrated in Fig 5y_1 The mass increment is clearly given by

$$\boldsymbol{\Delta M} \;=\; \boldsymbol{\rho}\,\boldsymbol{\Delta V} \tag{30}$$

This is the mass as measured by observers at both $\boldsymbol{X}$ and $\boldsymbol{A}$.

5x.3 Next we consider the same situation but with a variation of the energy scale. $\boldsymbol{\xi_X}$ is the value of the energy scale at location $\boldsymbol{X}$ and $\boldsymbol{\xi_A}$ the value at location $\boldsymbol{A}$. This is illustrated in Fig 5y_2.

For an observer with the mass at $\boldsymbol{X}$, we have

$$\boldsymbol{\Delta M_X^X} \;=\; \boldsymbol{\rho_X^X}\;\boldsymbol{\Delta V} \tag{31}$$

where $\boldsymbol{\Delta M_X^X}$ is the increment in mass at location $\boldsymbol{X}$ as measured by an observer at $\boldsymbol{X}$; $\boldsymbol{\rho_X^X}$ is the density at $\boldsymbol{X}$ as measured by an observer at $\boldsymbol{X}$.

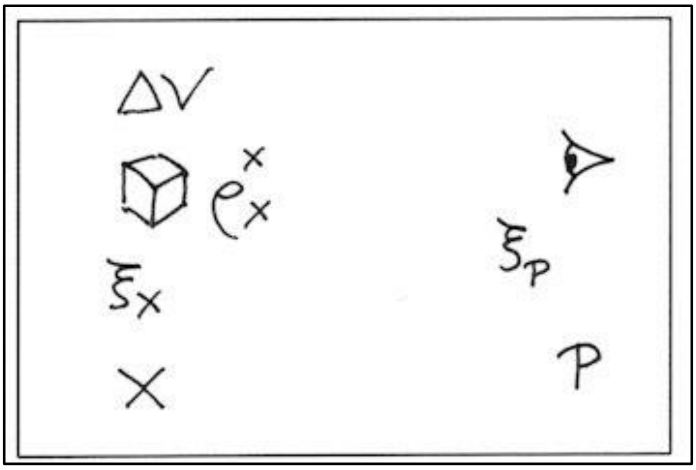

Fig 5y_2. Mass increment at location $\boldsymbol{X}$ and a remote observer at location $\boldsymbol{P}$, with a variation of the energy scale.

For a remote observer at location P, we have

$$\boldsymbol{\Delta M_X^A} \;=\; \boldsymbol{\rho_X^A} \; \boldsymbol{\Delta V} \tag{32}$$

where $\boldsymbol{\Delta M_X^A}$ is the increment in mass at location $\boldsymbol{X}$ as measured by an observer at $\boldsymbol{A}$; $\boldsymbol{\rho_X^A}$ is the density at $\boldsymbol{X}$ as measured by an observer at $\boldsymbol{A}$.

Our conjecture, equation (13), means

$$\boldsymbol{\xi_X \, \Delta M_X^X} \;=\; \boldsymbol{\xi_A \, \Delta M_X^A} \tag{33}$$

Hence

$$\boldsymbol{\Delta M_X^A} \;=\; \left(\frac{\boldsymbol{\xi_X}}{\boldsymbol{\xi_A}}\right) \boldsymbol{\Delta M_X^X} \tag{34}$$

and

$$\boldsymbol{\rho_X^A} \;=\; \left(\frac{\boldsymbol{\xi_X}}{\boldsymbol{\xi_A}}\right) \boldsymbol{\rho_X^X} \tag{35}$$

5x.4 We now consider the mass of an extended body enclosed in volume $\boldsymbol{V}$, as illustrated in Fig 5y_3. In this situation of no energy scale variation, the total mass is given by summing the individual increments within the volume, which is the same as integrating over the volume

$$\boldsymbol{M} \;=\; \sum \boldsymbol{\rho \, \Delta V} \;=\; \iiint \boldsymbol{\rho \, dV} \tag{36}$$

This mass is what is measured by observers X within the volume, or by remote observers A.

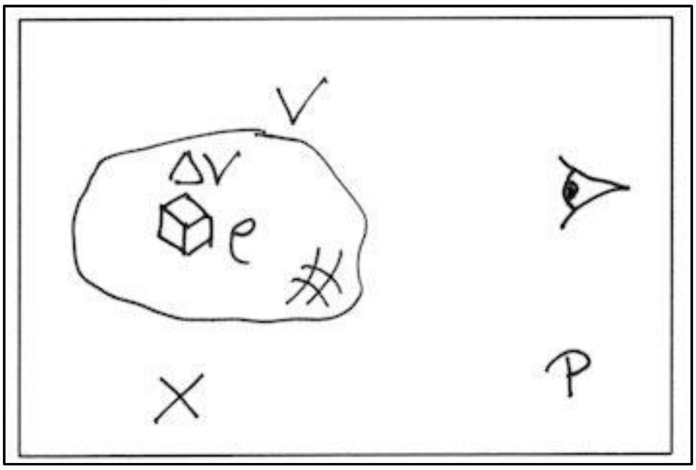

Fig 5y_3. Mass of an extended volume V, a volume element at location X and a remote observer at location P, with no variation of the energy scale.

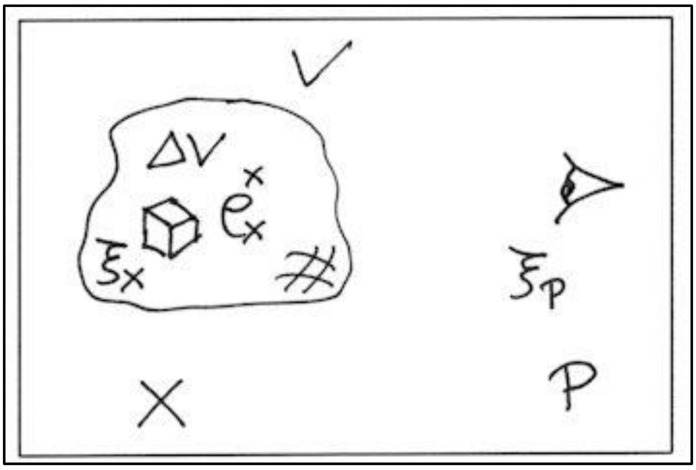

Fig 5y_4. Mass of an extended volume V, a volume element at location X and a remote observer at location P, with a variation of the energy scale.

5x.5 We now consider the mass of an extended body with a variation of the energy scale. This is where things become a little more complicated. The situation is illustrated in Fig 5y_4. Every element in the volume is different not only because the density can be different, but also because the value of the energy scale can be different. An observer at any location inside the volume does not see the "intrinsic" mass of the other elements, but only the mass modified by the variation of the energy scale. This means the apparent mass of every element appears different to every other element. We can no longer obtain the mass by considering a single observer at an internal point X and simply adding up the masses of the other elements.

5x.6 However, we can consider an external observer at remote location $\boldsymbol{A}$, for whom the total mass is given by

$$\xi_A\, M_V^A \;=\; \sum \xi_X\, \rho_X^X\, \Delta V \;=\; \iiint \xi_X\, \rho_X^X\, dV \tag{37}$$

where M_V^A is the mass of the volume V as measured by observer A . In this case the subscript V refers to the whole volume and not to a single location.

5x.7 We can now use equation (13) to get at the mass of the volume as measured by an observer inside the volume at X

$$\xi_X\, M_V^X \;=\; \xi_A\, M_V^A \tag{38}$$

where M_V^X is the mass of the volume V as measured by observer X.

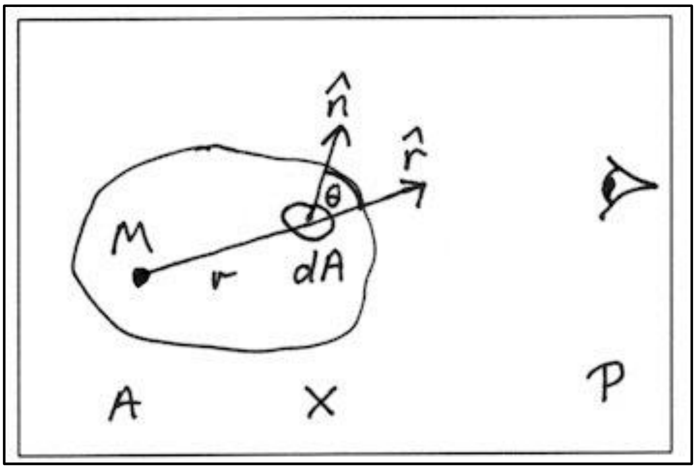

Fig 5y_5. Mass M at A inside a closed surface, surface increment dA at X, and a remote observer at location P, with no variation of the energy scale.

5x.8 **Gauss's law for gravity**

Gauss's law for gravity states (Wikipedia; Alonzo & Finn, p423) "the gravitational flux through any closed surface is proportional to the enclosed mass". We start with the situation of no variation of the energy scale, illustrated in Fig 5y_5. A mass M lies inside the closed surface a distance r away from surface element dA at location X. The gravitational field at X is

$$g \;=\; \frac{G\,M}{r^2} \tag{39}$$

The increment of area normal to the surface is

$$dA \;=\; \frac{r^2\, d\Omega}{\cos\theta} \tag{40}$$

The vector dot product of these is

$$\boldsymbol{g} \,.\, \boldsymbol{dA} \;=\; \frac{\boldsymbol{G\,M}}{\boldsymbol{r^2}} \cdot \frac{\boldsymbol{r^2\,d\Omega}}{\boldsymbol{\cos\theta}} \;=\; \boldsymbol{G\,M\,d\Omega} \tag{41}$$

The flux $\boldsymbol{F}$ is the integral of the dot product over the surface

$$\boldsymbol{F} \;=\; \iint \boldsymbol{g} \cdot \boldsymbol{dA} \;=\; \boldsymbol{G\,M} \iint \boldsymbol{d\Omega} \;=\; \boldsymbol{4\,\pi\,G\,M} \tag{42}$$

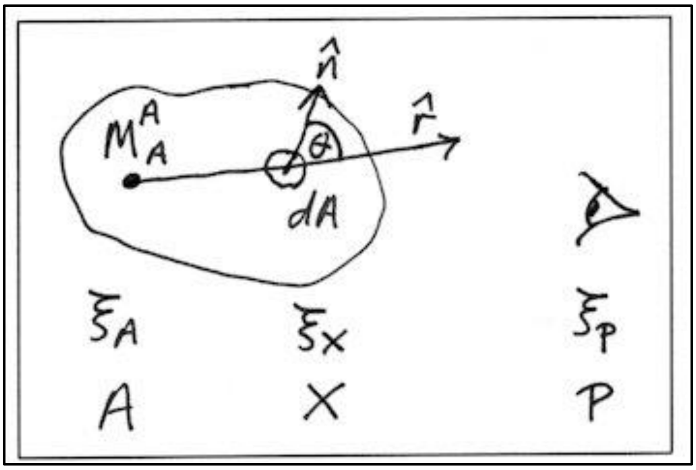

Fig 5y_5. Mass M at A inside a closed surface, surface increment dA at X, and a remote observer at location $\boldsymbol{P}$*, with a variation of the energy scale.*

5x.9 For our conjecture of variations of the energy scale we need to unpick Gauss's law. We consider a remote observer at remote location $\boldsymbol{P}$ measuring the gravitational flux through a small area of surface $\boldsymbol{dA}$ at location $\boldsymbol{X}$ arising from an element of mass $\boldsymbol{\Delta M}$ at location $\boldsymbol{A}$ within the surface.

$$\boldsymbol{\Delta F_X^P} \;=\; \frac{\boldsymbol{G_X^P\;\Delta M_A^P}}{\boldsymbol{r^2}} \cdot \frac{\boldsymbol{r^2\,\Delta\Omega}}{\boldsymbol{\cos\theta}} \;=\; \left(\frac{\boldsymbol{\xi_P}}{\boldsymbol{\xi_X}}\,\boldsymbol{G_X^X}\right)\left(\frac{\boldsymbol{\xi_A}}{\boldsymbol{\xi_P}}\,\boldsymbol{\rho_A^A\,\Delta V}\right)\boldsymbol{\Delta\Omega} \tag{43}$$

or

$$\boldsymbol{\Delta F_X^P} \;=\; \boldsymbol{G_X^X}\left(\frac{\boldsymbol{\xi_A}}{\boldsymbol{\xi_X}}\right)\boldsymbol{\rho_X^X\;\Delta V\,\Delta\Omega} \;=\; \boldsymbol{G\,\rho\,\Delta V}\left(\frac{\boldsymbol{\xi_A}}{\boldsymbol{\xi_X}}\right) \tag{44}$$

This result is somewhat similar to equation (23) above. To obtain the total flux we have to sum over all the mass increments inside the surface and then sum over all the angular elements of the surface. We cannot do this easily because of the two $\boldsymbol{\xi}$ factors.

5x.10 We can make some progress by considering a surface of constant ξ_S. When we do that and apply the result of equation (38) we arrive at the total flux across surface S as measured by a remote observer at P

$$F_S^P = 4\,\pi\,G\,\left(\frac{\xi_P}{\xi_S}\right)\,M_S^P \tag{45}$$

where M_S^P is the mass inside surface S as measured by P. This is our version of Gauss's law for gravitation.

6x. Potential Theory

6x.1 Newtonian gravity is generally taken to be covered by potential theory in that the gravitational field is a scalar potential field and the gravitational acceleration is the gradient of this scalar potential. We need to understand how our conjecture of energy scale variations changes the standard results of potential theory as applied to the gravitational field. What follows is based on Binney & Tremaine (2009), Potential Theory, section 2.1 General Results. The notation also follows Binney & Tremaine and is slightly different from previous sections.

6x.2 The gravitational acceleration at $\boldsymbol{x}$ arising from a distribution of matter in region $\boldsymbol{x'}$ is given by (Binney & Tremaine equation 2.2)

$$\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{G} \int \boldsymbol{d^3 x'} \frac{\boldsymbol{x'} - \boldsymbol{x}}{|\boldsymbol{x'} - \boldsymbol{x}|^3} \boldsymbol{\rho}(\boldsymbol{x'}) \tag{46}$$

For our conjecture of energy scale variations this becomes

$$\boldsymbol{\xi}(\boldsymbol{x})\, \boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{G} \int \frac{\boldsymbol{x'} - \boldsymbol{x}}{|\boldsymbol{x'} - \boldsymbol{x}|^3}\, \boldsymbol{\xi}(\boldsymbol{x'})\, \boldsymbol{\rho}(\boldsymbol{x'})\, \boldsymbol{d^3 x'} \tag{47}$$

6x.3 The gravitational potential is defined by (Binney & Tremaine equation 2.3)

$$\boldsymbol{\Phi}(\boldsymbol{x}) = -\boldsymbol{G} \int \boldsymbol{d^3 x'} \frac{\boldsymbol{1}}{|\boldsymbol{x'} - \boldsymbol{x}|} \boldsymbol{\rho}(\boldsymbol{x'}) \tag{48}$$

For our conjecture of energy scale variations this becomes

$$\boldsymbol{\xi}(\boldsymbol{x})\, \boldsymbol{\Phi}(\boldsymbol{x}) = -\boldsymbol{G} \int \frac{\boldsymbol{1}}{|\boldsymbol{x'} - \boldsymbol{x}|}\, \boldsymbol{\xi}(\boldsymbol{x'})\, \boldsymbol{\rho}(\boldsymbol{x'})\, \boldsymbol{d^3 x'} \tag{49}$$

6x.4 Binney & Tremaine give the standard result (their equation 2.4) that differentiating with respect to $\boldsymbol{x}$ (not $\boldsymbol{x'}$)

$$\boldsymbol{\nabla}_x \left(\frac{\boldsymbol{1}}{|\boldsymbol{x'} - \boldsymbol{x}|} \right) = \frac{\boldsymbol{x'} - \boldsymbol{x}}{|\boldsymbol{x'} - \boldsymbol{x}|^3} \tag{50}$$

6x.5 Applying equation (50) to equation (48) leads to the standard result

$$\boldsymbol{g}(\boldsymbol{x}) = -\boldsymbol{\nabla} \{\boldsymbol{\Phi}(\boldsymbol{x})\} \tag{51}$$

For our conjecture of energy scale variations and applying equation (50) to equation (49) this becomes

$$\boldsymbol{\xi}(\boldsymbol{x})\, \boldsymbol{g}(\boldsymbol{x}) = -\boldsymbol{\nabla} \{\boldsymbol{\xi}(\boldsymbol{x})\ \boldsymbol{\Phi}(\boldsymbol{x})\} \tag{52}$$

6x.6 Binney & Tremaine also show that taking the divergence of equation (45) leads to Poisson's Equation

$$\boldsymbol{\nabla}^2\{\boldsymbol{\Phi}(\boldsymbol{x})\} = \boldsymbol{\nabla}\cdot\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{4\pi G}\,\boldsymbol{\rho}(\boldsymbol{x}) \tag{53}$$

For our conjecture of energy scale variations this becomes

$$\boldsymbol{\nabla}^2\{\boldsymbol{\xi}(\boldsymbol{x})\,\boldsymbol{\Phi}(\boldsymbol{x})\} = \boldsymbol{\nabla}\cdot\{\boldsymbol{\xi}(\boldsymbol{x})\,\boldsymbol{g}(\boldsymbol{x})\} = \boldsymbol{4\pi G}\,\{\boldsymbol{\xi}(\boldsymbol{x})\,\boldsymbol{\rho}(\boldsymbol{x})\} \tag{54}$$

6x.7 In trying to understand galaxy rotation curves it has become common practice to measure the gravitational acceleration (and hence the expected rotational velocity) by solving Poisson's Equation (equation (53)). If we can use observations to measure the density distribution, then we can solve Poisson's equation for the gravitational potential and differentiate this for the gravitational acceleration. However, in the context of energy scale variations this is no longer possible, because we have to solve equation (54) and we do not know the form of function $\boldsymbol{\xi}(\boldsymbol{x})$.

6x.8 We can now rewrite the standard equations using the subscript notation of previous sections. And we remember that subscripts are only needed for quantities with units involving energy; they are not required for distance, acceleration, etc. So for an observer at location $\boldsymbol{P}$, measuring the gravity at location $\boldsymbol{P}$, arising from a distribution of matter at locations $\boldsymbol{X}$, and replacing $\boldsymbol{x'}$-$\boldsymbol{x}$ with $\boldsymbol{r}$

Gravitational acceleration, from equation (47)

$$\boldsymbol{\xi_P\,g} = \boldsymbol{\xi_P\,\ddot{r}} = -\boldsymbol{G}\int\frac{\boldsymbol{1}}{\boldsymbol{r}^2}\,\boldsymbol{\xi_X}\;\boldsymbol{\rho_X^X}\;\boldsymbol{dV} \tag{55}$$

Gravitational potential, from equation (49)

$$\boldsymbol{\xi_P\,\Phi} = -\boldsymbol{G}\int\frac{\boldsymbol{1}}{\boldsymbol{r}}\,\boldsymbol{\xi_X\,\rho_X^X}\;\boldsymbol{dV} \tag{56}$$

Gradient of scalar potential, equation (50)

$$\boldsymbol{\xi_P\,g} = \boldsymbol{\xi_P\,\ddot{r}} = -\boldsymbol{\nabla}\,\{\boldsymbol{\xi_P\,\Phi}\} \tag{57}$$

Poisson's Equation, equation (53)

$$\boldsymbol{\nabla}^2\{\boldsymbol{\xi_P\,\Phi}\} = \boldsymbol{4\pi G}\,\{\boldsymbol{\xi_P\,\rho_P^P}\} \tag{58}$$

6x.9 Finally, for a spherically-symmetric distribution of both matter $\boldsymbol{\rho}$ and energy scale variation $\boldsymbol{\xi}$, the radial acceleration from equation (55) becomes

$$\xi_P\,\ddot{r} \;=\; -\,\frac{G}{r^2}\int_0^r \xi_X\, dM_X^X \tag{59}$$

where dM_X^X is the mass of the shell at distance X from the centre, and r is the distance of P from the centre.

6x.10 In summary, for our conjecture of variations of the energy scale, we replace some quantities with new versions multiplied by our energy scale factor, ξ:

(a) for the gravitational potential

$$\Phi(r) \;\rightarrow\; \xi_P(r)\;\Phi_X^P(r) \tag{60}$$

where $\Phi_X^A(r)$ is the gravitational potential at X as measured by an observer at P.

(b) for the density

$$\rho(r) \;\rightarrow\; \xi_P(r)\;\rho_X^P(r) \tag{61}$$

where $\rho_X^P(r)$ is the density at X as measured by an observer at P.
and where we've added the "(r)" to remind ourselves that these are functions of position.

7x. Baryonic mass and dynamical mass

7x.1 In many astronomical scenarios astronomers talk about the baryonic mass of a system and the dynamical mass of a system. These are two different measures of what should be the same quantity. However, the measures often differ with the dynamical mass being around five times greater than the baryonic mass. We look at these two measures through the example of a galaxy cluster, where up to a thousand galaxies form a bound system often embedded in a large halo of X-ray emitting gas.

7x.2 The baryonic mass of an object is the amount of matter that we observe to be present. For a galaxy cluster we can observe the light from the individual galaxy members and, using a mass-to-light ratio, add up the total mass of all the galaxies. We can also observe the X-ray emission from the hot gas and again arrive at a total mass for the gas present. Usually the mass of the gas exceeds the mass of the galaxies by at least a factor of ten. The baryonic mass of the cluster is then the sum of the mass of the galaxy members and the mass of the gas.

7x.3 The dynamical mass of an object is the amount of mass the object needs to have in order to explain its dynamic properties. For a galaxy cluster the velocities of the galaxy members lead to a dynamic mass for the whole cluster based on the virial theorem. The virial mass is the mass required to keep the galaxies together as a bound system. If we assume the X-ray gas is in hydrodynamic equlibrium then calculations tell us how much mass is needed for gravity to balance the gas pressure. Finally, weak gravitational lensing of remote objects by the galaxy cluster lead to another independent measure of the dynamical mass. These three measures of the dynamical mass are usually in good agreement with one another, agreeing with one another to within a factor of two.

7x.4 The discrepancy between the smaller baryonic mass and the larger dynamical mass is using explained by the existence of large amounts of dark matter. By adding in five times as much dark matter as baryonic matter, there is sufficient gravity to explain away the observations. A minority of astronomers also consider modifications to the law of gravity. The best known of these is MOND, modified Newtonian dynamics, introducted by Milgrom (1963). Here Newton's law of gravity holds in the high acceleration regime but switches to a modified law in the low acceleration regime. The switchover acceleration is around 1.0×10^{-10} m/s^2.

7x.5 Our conjecture of variations of the energy scale gives us a completely different explanation. The effect of a remote mass at $\boldsymbol{X}$ on location $\boldsymbol{P}$ is given by equation (14)

$$\boldsymbol{M_X^P} = \left(\frac{\boldsymbol{\xi_X}}{\boldsymbol{\xi_P}}\right) \boldsymbol{M_X^X} \tag{62}$$

where $\boldsymbol{M_X^X}$ is the baryonic mass, and $\boldsymbol{M_X^P}$ is the dynamical mass. All we need is for the ratio of the two $\boldsymbol{\xi}$ values to be sufficiently large (around 5) and the mass discrepancy goes away.

7x.6 For a spherically-symmetric distribution of matter the dynamical mass is given in terms of the baryonic mass by (following equation (58))

$$\boldsymbol{M_{dyn}} = \boldsymbol{M_r^r} = \frac{\mathbf{1}}{\boldsymbol{\xi_r}} \int_{\mathbf{0}}^{\boldsymbol{r}} \boldsymbol{\xi_X}\, \boldsymbol{dM_X^X} \tag{63}$$

This is the mass that we use in Newton's law of gravity.
Each shell of matter is weighted by the value of our $\boldsymbol{\xi}$-function on that shell, and the whole sum is then divided by the value of the $\boldsymbol{\xi}$-function at the point in question.
Our conjecture of variations of the energy scale means we are altering the effective mass. We are not introducing any exotic forms of matter in the form of dark matter, and we are not changing the law of gravity.

8x. Another look at galaxy rotation curves

8x.1 We can now revisit galaxy rotation curves to see how our new work on mass and potential theory fits in. We start by showing that relativistic effects can be ignored and then note that we can use the equations for spherical symmetry even though spiral galaxies are clearly not spherical.

8x.2 It is well-known that the effects of the Special Theory of Relativity (SR) can be ignored when the velocity, $\boldsymbol{v}$, is much less than the speed of light, $\boldsymbol{c}$, (Schutz, 2009)

$$\frac{\boldsymbol{v}}{\boldsymbol{c}} \ll \mathbf{1} \tag{61}$$

This also means that the $\boldsymbol{\gamma}$ factor of SR is very close to 1.

$$\boldsymbol{\gamma} = \sqrt{\mathbf{1} - \left(\frac{\boldsymbol{v}^2}{\boldsymbol{c}^2}\right)} \approx \mathbf{1} \tag{62}$$

The rotational velocity of the stars & gas in a spiral galaxy rarely exceed 300 km/s or 0.1% of the speed of light. For example, Fig 1 shows NGC 2403 has a maximum rotational velocity of just below 140 km/s, leading to (rewriting equation (1))

$$\frac{\boldsymbol{v}^2}{\boldsymbol{c}^2} \sim \mathbf{10^{-6}} \tag{63}$$

We can safely conclude that the effects of SR play no part in the rotation curves of spiral galaxies.

8x.3 It is also well-known that the effects of the General Theory of Relativity (GR) can be ignored when (Schutz, 2009)

$$\frac{\boldsymbol{G\,M}}{\boldsymbol{R\,c^2}} \ll \mathbf{1} \tag{64}$$

For our galaxy: the mass, $\boldsymbol{M}$, is around 2×10^{41} kg (10^{11} solar masses); a typical distance to the spiral arms, $\boldsymbol{R}$, is around 5×10^{20} m (15 kpc); leading to

$$\frac{\boldsymbol{G\,M}}{\boldsymbol{R\,c^2}} \sim \mathbf{3 \times 10^{-7}} \tag{65}$$

We can safely conclude that the effects of GR play no part in the rotation curves of spiral galaxies.

8x.4 Equation (1), in section 2x.3 above, can be written as

$$\frac{v^2}{c^2} = \frac{G\,M(r)}{r\;c^2} \tag{66}$$

If the left hand side is small, then Special Relativity (SR) can be ignored; as mentioned in 2x.3 above. If the right hand side is small, then General Relativity (GR) can be ignored; as mentioned in 2x.4 above. It is interesting that the equation for the rotational velocity of spiral galaxies contains both the conditions that special relativity and general relativity can be ignored.

8x.5 Spiral galaxies are not spherically symmetric; they are flattened disks. However, most of the baryonic mass is concentrated in a central spherical bulge. And the density of the stars & gas in the spiral arms drops off very rapidly with distance from the centre. These two conditions mean that if we work with the equations of spherical symmetry, then the errors we introduce are less than 5% (Binney & Tremaine, 2008; Bovy, 2023).

8x.6 We are assuming, of course, that there is no dark matter; so no wimps, no axions, no sterile neutrinos or whatever. This means that the matter we observe is all there is.
For disk galaxies the light from the stars leads to a measure of the surface density, after applying a mass-to-light ratio. Radio measurements of neutral hydrogen gas support these, as do measurements of molecular hydrogen. All three are in good agreement and show that the density falls of exponentially away from the galaxy centre (Bovy, 2023).
Spectrographic observations provide radial velocities, leading to the rotation curve. So, the data we have to work with is a set of surface density and velocity measurements.

8x.7 For a disk galaxy we know the radial acceleration is given by

$$\ddot{r} = -\frac{v^2}{r} \tag{67}$$

and the increment of mass at distance x as measured by an observer at X is

$$dM_X^X = 4\,\pi\,\sigma_X^X\,x\,dx \tag{67}$$

where σ_X^X is the surface density.

8x.8 From our work on potential theory, we can now apply equation (58) to give

$$-\,\xi_r\,\ddot{r} = \xi_r\frac{v^2}{r} = \frac{G}{r^2}\int_0^r \xi_X\,dM_X^X \tag{68}$$

or

$$\boldsymbol{\xi_r \;=\; \frac{G}{r\,v^2}\int_0^r \xi_X\; dM_X^X} \tag{69}$$

8x.9 To solve this we split the integral into two parts

$$\boldsymbol{\xi_r \;=\; \frac{G}{r\,v^2}\left\{\int_0^R \xi_X\; dM_X^X \;+\; \int_R^r \xi_X\; dM_X^X\right\}} \tag{70}$$

We choose R to be close to the galaxy centre, where we can approximate the first integral as

$$\boldsymbol{\int_0^R \xi_X\; dM_X^X \;\approx\; \xi_A \int_0^R dM_X^X} \tag{71}$$

where ξ_A is the average value of the ξ function in the central region. This leads to

$$\boldsymbol{\xi_R \;=\; \xi_A\;\frac{G}{R\,v(R)^2}\int_0^R dM_X^X} \tag{72}$$

Having fixed the value of ξ at one point, we can then use equation (70) to determine the value at the next point and so gradually integrate outwards.

8x.10 At the end of this process we have values of the ξ across the galaxy. If we so wish we can treat these values as first approximations and repeat the process to obtain better approximations, including dropping the assumption of spherical symmetry and working with the full expressions for axial symmetry.
This work, as applied to the SPARC galaxies, is described in detail in viXra paper 1903.0109 (JoKe 2019).

Cx. General relativity

Cx.1 In Einstein's general theory of relativity, gravity arises from the curvature of spacetime, which is defined by the metric tensor. In the weak-field approximation, the metric tensor is described by the line element (Schutz, 1985)

$$ds^2 = -(1 - 2\,\chi)dt^2 + (1 + 2\,\chi)(dx^2 + dy^2 + dz^2) \tag{70}$$

where

$$\chi = \frac{G\,M}{r\,c^2} \tag{42}$$

and χ is associated with the Newtonian gravitational potential, φ,

$$\varphi = -\frac{G\,M}{r} \tag{43}$$

and M is the remote mass.

Cx.2 Equation (42) arises because the metric, equation (41), leads to the radial acceleration being given by (Schutz, 1985)

$$\ddot{r} = c^2 \frac{\partial \chi}{\partial r} = -\frac{G\,M}{r^2} = -\nabla\varphi \tag{44}$$

Cx.3 For our conjecture of energy scale variations, we require the gravitational acceleration, at location X, arising from mass M at location A, to be given by

$$\ddot{r} = -\nabla\varphi = -\frac{G\,M}{r^2}\left(\frac{\xi_A}{\xi_X}\right) \tag{51}$$

It is clear from sections 4x.2, 4x.3 & 4x.4 (above) that this expression for the acceleration is independent of the location of the observer, i.e. all observers will agree on its value. This is because equation (51) depends on the value of the ξ function at X, ξ_X, and at the remote mass, ξ_A, but not on the value at the observer.

Cx.4 Equation (51) implies the gravitational potential is given by

$$\varphi = -G\,M\,\xi_A \int_{\infty}^{X} \frac{dr}{r^2\,\xi_r} \tag{45}$$

We cannot integrate this as ξ_r is an unknown function of the radial distance, r. However, this is not normally a problem as it is usually the gradient of the potential that we need and not the potential itself.

Cx.5 To summarise, for our conjecture of energy scale variations, our weak-field metric is given by

$$\boldsymbol{ds^2 = -(1 - 2\,\chi)dt^2 + (1 + 2\,\chi)(dx^2 + dy^2 + dz^2)} \tag{46}$$

where

$$\boldsymbol{\chi = -\frac{G\,M\,\xi_A}{c^2}\int_{\infty}^{X}\frac{dr}{r^2\,\xi_r}} \tag{47}$$

Cx.6 Equation (47) shows that we are introducing a new dimensionless scalar field, $\boldsymbol{\xi}$. This field defines the strength of the energy scale and how it varies from location to location.

Fx. Ideal gas

Fx.1 It is instructive to look at an ideal gas that is spread across two regions having different values for the energy scale. The situation is illustrated in Fig 5.1.

Fx.2 We consider two gas filled regions, A & B, that differ only in the values of the energy scale, ξ_A & ξ_B respectively. We also have a remote observer at X. It is only the energy scale that varies so the length and time scales are the same in both regions.

Fx.3 For the two regions we assume
a) the volumes are the same
b) the intrinsic masses of the gas molecules are the same
c) the average velocities of the gas molecules are the same

Fx.4 The usual expression for the kinetic energy of the gas is

$$\boldsymbol{E = \frac{1}{2}\, m\, v^2 \;=\; \frac{3}{2}\, k\, T} \tag{48}$$

where v is the average velocity of the gas molecules; k is Boltzmann's constant. Boltzmann's constant, k, (1.398×10^{-23} J K^{-1}) has units of energy per degree and so

$$\boldsymbol{k_A^X \;=\; k_A^A \left(\frac{\xi_A}{\xi_X}\right) \;=\; k \left(\frac{\xi_A}{\xi_X}\right)} \tag{49}$$

Fx.5 For our conjecture of energy scale variations, the observer at X measures the kinetic energy in region A as

$$\boldsymbol{E_X^A = \frac{1}{2}\; m_X^A\; v^2 \;=\; \frac{1}{2}\, m\, v^2 \left(\frac{\xi_A}{\xi_X}\right)} \tag{50}$$

$$\boldsymbol{=\; \frac{3}{2}\; k_X^A\; T_A^X = \; \frac{3}{2}\, k\, T_A^X \left(\frac{\xi_A}{\xi_X}\right)} \tag{51}$$

Fx.6 Similarly for region B

$$\boldsymbol{E_X^B} \; = \; \frac{\mathbf{1}}{\mathbf{2}}\, \boldsymbol{m\, v^2} \left(\frac{\boldsymbol{\xi_B}}{\boldsymbol{\xi_X}}\right) \; = \; \frac{\mathbf{3}}{\mathbf{2}}\, \boldsymbol{k\, T_B^X} \left(\frac{\boldsymbol{\xi_B}}{\boldsymbol{\xi_X}}\right) \tag{52}$$

Fx.7 Comparing equations (50), (51), (52) it is clear that all observers measure exactly the same temperature

$$\boldsymbol{T_A^X} \; = \; \boldsymbol{T_B^X} \; = \boldsymbol{T} \tag{53}$$

Fx.8 The usual expression for the ideal gas law is

$$\boldsymbol{p\, V} \; = \; \boldsymbol{n\, R\, T} \; = \boldsymbol{n\, A_N\, k\, T} \tag{54}$$

where $\boldsymbol{n}$ is the amount of gas in mols; $\boldsymbol{R}$ the gas constant; $\boldsymbol{A_N}$is Avogadro's number

Fx.9 For region $\boldsymbol{A}$, observer $\boldsymbol{X}$ measures

$$\boldsymbol{p_X^A\, V_X^A} \; = \boldsymbol{n\, A_N\, k_X^A\, T_X^A} \tag{55}$$

or

$$\boldsymbol{p_A^A} \left(\frac{\boldsymbol{\xi_A}}{\boldsymbol{\xi_X}}\right) \boldsymbol{V} \; = \boldsymbol{n\, A_N\, k} \left(\frac{\boldsymbol{\xi_A}}{\boldsymbol{\xi_X}}\right) \boldsymbol{T} \tag{56}$$

or

$$\boldsymbol{p_A^A\, V} \; = \boldsymbol{n\, A_N\, k\;\, T} \tag{64}$$

Fx.10 Similarly for region $\boldsymbol{B}$

$$\boldsymbol{p_B^B\, V} \; = \boldsymbol{n\, A_N\, k\;\, T} \tag{65}$$

So, we end up with

$$\boldsymbol{p_A^A} \; = \; \boldsymbol{p_B^B} = \boldsymbol{p} \tag{57}$$

This means that observers in the two regions measure the same pressure. This is, of course, as it must be as we assumed the same mass and average velocity in both regions to begin with.

Fx.11 The observer at X measures the same temperature in both regions, but different pressures (energy densities)

$$\boldsymbol{p_X^A} = \boldsymbol{p_A^A} \left(\frac{\xi_A}{\xi_X}\right) = \boldsymbol{p} \left(\frac{\xi_A}{\xi_X}\right) \tag{58}$$

$$\boldsymbol{p_X^B} = \boldsymbol{p_B^B} \left(\frac{\xi_B}{\xi_X}\right) = \boldsymbol{p} \left(\frac{\xi_B}{\xi_X}\right) \tag{59}$$

The different pressures are balanced by the different values for Boltzmann's constant. Hence, there is no flow of matter from high pressure to low pressure. Again, this also follows from the gas molecules having the same average velocity in both regions.

Jx. Movement of mass

Jx.1 We can clarify our ideas of variations of the energy scale by considering what happens when a mass moves from one location to another.

Jx.2 **Case 1**

We consider the variation of the energy scale to be attached to space. This means every mass adopts the value of the energy scale at its location; the mass has no effect on this value. We consider a mass $\boldsymbol{m}$ initially at location $\boldsymbol{X}$ that moves to location $\boldsymbol{Y}$, as measured by an observer at $\boldsymbol{A}$.

Jx.3 Observer $\boldsymbol{A}$ measures the initial mass $\boldsymbol{T_i}$ as

$$\boldsymbol{T_i} = \left(\frac{\xi_X}{\xi_A}\right) \boldsymbol{m_X^X} = \left(\frac{\xi_X}{\xi_A}\right) \boldsymbol{m} \qquad (60)$$

and the final mass $\boldsymbol{T_f}$ as

$$\boldsymbol{T_f} = \left(\frac{\xi_Y}{\xi_A}\right) \boldsymbol{m_Y^Y} = \left(\frac{\xi_Y}{\xi_A}\right) \boldsymbol{m} \qquad (70)$$

Jx.4 For conservation of mass to hold, the initial and final masses must be the same. Clearly, this is only possible if

$$\xi_X = \xi_Y \qquad (71)$$

i.e. the value of the energy scale at both locations must be the same, which means the energy scale cannot vary from location to location.

Jx.5 This simple example shows us that variations of the energy scale cannot be attached to space-time locations.

Jx.6 **Case 2**

Next, we consider the value of the energy scale to be attached to the mass rather than the location. This means that when the mass moves the value of the energy scale moves with it. We consider exactly the same situation as before. A mass $\boldsymbol{m}$ initially at location $\boldsymbol{X}$ that moves to location $\boldsymbol{Y}$, as measured by an observer at $\boldsymbol{A}$.

Jx.7 We end up with exactly the same equations. The initial mass is given by equation (?6?), and the final mass by equation (60). For conservation of mass to hold, we also end up with equation (70). However, now our interpretation is different. The value of the ξ function at Y is the same as that at X because the mass m has taken it there.

Jx.8 This example shows us that our conjecture of variations in the energy scale can work, provided the value of the energy scale is attached to the energy (mass) itself and not to the the location.

Jx.9 **Case 3**
We now consider what happens when a small mass moves between two other masses. Consider a mass M at location X, a mass N at location Y, and our small mass m that moves from X. to Y.

Jx.10 Our observer at location A measures the initial total mass T_i as

$$\boldsymbol{T_i} = \left(\frac{\xi_X}{\xi_A}\right) \boldsymbol{M} + \left(\frac{\xi_Y}{\xi_A}\right) \boldsymbol{N} + \left(\frac{\xi_X}{\xi_A}\right) \boldsymbol{m} \tag{72}$$

Jx.11 We know, from Case 2, that the value of the energy scale for mass M at location X does not change when the small mass moves. However, the same is not true for mass N at location Y when the small mass is mixed in.
Our observer measures the final mass as

$$\boldsymbol{T_f} = \left(\frac{\xi_X}{\xi_A}\right) \boldsymbol{M} + \left(\frac{\xi_{Y'}}{\xi_A}\right) \boldsymbol{N} + \left(\frac{\xi_{Y'}}{\xi_A}\right) \boldsymbol{m} \tag{73}$$

where the primes indicate the final state values.

Jx.12 We demand that conservation of mass holds. So

$$\left(\frac{\xi_Y}{\xi_A}\right) \boldsymbol{N} + \left(\frac{\xi_X}{\xi_A}\right) \boldsymbol{m} = \left(\frac{\xi_{Y'}}{\xi_A}\right) \boldsymbol{N} + \left(\frac{\xi_{Y'}}{\xi_A}\right) \boldsymbol{m} \tag{74}$$

or

$$\xi_{Y'} = \xi_Y - (\xi_Y - \xi_X)\left(\frac{\boldsymbol{m}}{\boldsymbol{N}+\boldsymbol{m}}\right) \tag{75}$$

This demonstrates that our conjecture of variations in the energy scale continues to work when other masses are involved. Equation (75) gives us the rule to calculate the new value of the energy scale when masses with different values combine.

Jx.13 **Case 4**

We are now in the position where the value of the energy scale at a location is determined by the mass at that location. However, there is an immediate problem as to what we do when there is no mass present at a location. The value cannot be zero as this would lead to infinities in equations (72) and (73). Instead, we consider a background value, $\boldsymbol{\xi_b}$.

Jx.14 Our observer now measures an initial mass of

$$\boldsymbol{T_i} = \left(\frac{\xi_X + \xi_b}{\xi_A + \xi_b}\right) \boldsymbol{M} + \left(\frac{\xi_Y + \xi_b}{\xi_A + \xi_b}\right) \boldsymbol{N} + \left(\frac{\xi_X + \xi_b}{\xi_A + \xi_b}\right) \boldsymbol{m} \tag{76}$$

and a final mass of

$$\boldsymbol{T_f} = \left(\frac{\xi_X + \xi_b}{\xi_A + \xi_b}\right) \boldsymbol{M} + \left(\frac{\xi_{Y'} + \xi_b}{\xi_A + \xi_b}\right) \boldsymbol{N} + \left(\frac{\xi_{Y'} + \xi_b}{\xi_A + \xi_b}\right) \boldsymbol{m} \tag{77}$$

where the primes indicate the final state values.

Jx.15 We again impose conservation of mass. So

$$\left(\frac{\xi_Y + \xi_b}{\xi_A + \xi_b}\right) \boldsymbol{N} + \left(\frac{\xi_X + \xi_b}{\xi_A + \xi_b}\right) \boldsymbol{m} = \left(\frac{\xi_{Y'} + \xi_b}{\xi_A + \xi_b}\right) \boldsymbol{N} + \left(\frac{\xi_{Y'} + \xi_b}{\xi_A + \xi_b}\right) \boldsymbol{m} \tag{78}$$

or

$$\xi_{Y'} = \xi_Y - (\xi_Y - \xi_X) \left(\frac{\boldsymbol{m}}{\boldsymbol{N} + \boldsymbol{m}}\right) \tag{79}$$

So, the background value of the energy scale cancels out and plays no part in equation (79).

Jx.16 Equation (79) shows that if a mass with a low value of the energy scale moves to a mass with a high value, then the high value is reduced. Similarly, when a high value mass moves to a low value, then the low value is increased. So, over time we expect the highs and lows to get smoothed out.

Jx.17 Equation (78) shows us that the 'effective' mass is unchanged when matter moves from one region to another. If we have a large mass with a high energy scale value that is attracting material, then it continues to do so. Even though its high value is being eroded, its pulling power is not diminished. This is because the lowering of its ξ value is exactly balanced by the increase in mass, with the increase in the ξ value of the infalling material. So, once a region starts sucking in material then it continues to do so; small galaxies naturally grow into large galaxies.

Mx. Miscellaneous

Mx.1 Energy Scale Variations and General Relativity

The metric tensor contains the gravitational potential, $\boldsymbol{\Phi}$, for the remote mass, $\boldsymbol{M}$.
The form of $\boldsymbol{\Phi}$ is not specified but often Newton's formula for it is used

$$\boldsymbol{\Phi} = -\frac{\boldsymbol{G}\,\boldsymbol{M}}{\boldsymbol{r}} \tag{80}$$

Mx.2 Re-examine Poisson's Equation

$$\boldsymbol{\nabla}^2\boldsymbol{\Phi} = \mathbf{4}\,\boldsymbol{\pi}\,\boldsymbol{G}\,\boldsymbol{\rho} \tag{81}$$

In an expanding Universe an observer does not see a uniform density. Distant regions appear to have a higher density.
The gravitation potential at the observer decreases as the Universe expands and ...

Mx.3 FLRW metric

How far can we get assuming just the FLRW metric? FLRW means the geometry of space is homogeneous and isotropic.
We should not work with velocities (or fractions of the speed of light) because the galaxies are not speeding away from one another; it is space that is expanding (with time).
The CMB (cosmic microwave background) identifies the temperature when electrons & protons combined to form neutral hydrogen. The temperature was around 3,000K. The CMB today has a temperature of 2.7K meaning the Universe has expanded by a linear factor of 1100 (volume by 1.3 billion).
The uniformity of the CMB shows the Universe has expanded by the same amount in all directions, i.e. has expanded isotropically. This strongly suggests (but does not prove) that the expansion is also homogeneous.

Mx.4 Welcome Back to the Museum of Dark Matter

Space-time is described well by the FLRW metric.
Is there some underlying principle that drives the Universe to follow the FLRW metric?
Is the Friedmann Equation correct, i.e. does the energy-momentum tensor control the growth of the scale factor? Or is something else involved?

Mx.5 Space-time

Space-time, i.e. space & time, is described well by the FLRW metric.
Space appears to be both homogeneous and isotropic, i.e. the geometry is both homogeneous and isotropic.
It is clear, from observations of objects such as planets, stars & galaxies, that the density of space is neither homogeneous nor isotropic. However, it is claimed, that

if we go to large enough scales (~100Mpc), then the density can be treated as isotropic & homogeneous.

Observations (what observations?) are consistent with the geometry of space being flat and Euclidean. If we could construct a large enough triangle across the Universe, then we would find the sum of the angles is 180 degrees.

The observed time dilation vs redshift relation is consistent with both GR (expansion of space) and SR (redshift => velocity)

$$\boldsymbol{\Delta t_o = (1 + z)\, \Delta t_e} \tag{82}$$

where Δt_o is the observed time interval, and Δt_e is the expected time interval.

12x. Discussion

12x.1 ??

13x. References

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