On the variation of the energy scale 26

Cosmology with no dark matter

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Summary

The ΛCDM model of cosmology is very successful at explaining the evolution of the Universe. However, it depends on the existence of cold dark matter (CDM) which, despite considerable effort, has never been detected. If dark matter does not exist, then there are serious implications for cosmology: the ratio of matter-to-radiation changes; the evolution of the Universe changes; the characteristics of the cosmic microwave background change. This paper discusses how the variation of the energy scale can provide an alternative to dark matter, and can mitigate against all the implications for cosmology of there being no dark matter.

1 Introduction

1.1 The best model we have for explaining the evolution of the Universe is the ΛCDM model (Λ=cosmological constant, CDM=cold dark matter). This assumes the Universe is made up of four components:

a) radiation (photons and neutrinos)
b) baryonic matter
c) cold dark matter (CDM)
d) cosmological constant (Λ).

It also assumes the Universe is flat, which means the energy density is the critical energy density.

1.2 The Friedmann Equation for such a Universe is

$$H^2 = \left\{\frac{\dot{a}}{a}\right\}^2 = \frac{8\,\pi\,G}{3\,c^2}\,\{\varepsilon_r + (\varepsilon_b + \varepsilon_d) + \varepsilon_\Lambda\} = \frac{8\,\pi\,G}{3\,c^2}\,\varepsilon_c \tag{1}$$

where

$$\varepsilon_b + \varepsilon_d = \varepsilon_m \tag{2}$$

where H is the Hubble constant; a the scale factor; ε_r the energy density of radiation; ε_b the energy density of baryonic matter; ε_d the energy density of dark matter; ε_Λ the energy density of a cosmological constant; ε_m the energy density of matter; ε_c the critical energy density of radiation.

1.3 The evolution of the Universe clearly changes if there is no dark matter, because matter is then made up of just the baryonic matter, and there is no ε_d term in equations (1) & (2). The energy density of matter is much smaller, and the balance between radiation and matter swings towards radiation.

1.4 In this paper we look at the consequences of there being no dark matter and how these can be mitigated against by allowing for variations in the energy scale. First we look at a few items that constrain all hypotheses for the evolution of the Universe, namely: baryogenesis; type Ia supernovae; the early Universe up to the cosmic microwave background.

1.5 Second we will look at the consequences of there being no dark matter. Third we look at two ways in which the Friedmann Equation can be modified to compensate for there being no dark matter. We concentrate on the consequences that affect observations of the cosmic microwave background and type Ia supernovae.

2 Big Bang Nucleosynthesis

2.1 Big Bang Nucleosynthesis (BBN) occurred around 200 seconds after the Big Bang when the light elements up to beryllium (^{8}Be) were created out of the existing protons and neutrons (Weinberg, 2008; Ryden 2017). Observations of the primordial abundances of hydrogen (^{1}H), deuterium (^{2}D), helium (^{3}He), and lithium (^{7}Li), fix the baryon-to-photon ratio, η, at

$$\eta = 6.1 \times 10^{-10} \tag{3}$$

2.2 Arguments presented in Weinberg (2008), Ryden (2017) (and elsewhere), show that the combination of

a) the baryon-to-photon ratio, η,
b) the Hubble constant, ~68 km/s/Mpc, and
c) the current temperature of the cosmic microwave background, 2.73 K,

leads directly to the current energy density of baryons, $\varepsilon_{b,0}$, being only 4.8% of the critical energy density, $\varepsilon_{c,0}$, required for a flat Universe

$$\Omega_{b,0} = \frac{\varepsilon_{b,0}}{\varepsilon_{c,0}} = 0.048 \tag{4}$$

where $\Omega_{b,0}$ is the current density parameter for baryons; $\varepsilon_{b,0}$ the current energy density of baryons; $\varepsilon_{c,0}$ the current critical energy density.

2.3 This number is independent of the existence of both dark matter and a cosmological constant. It places a solid constraint, not only on the ΛCDM model, but on all cosmological models and hypotheses that attempt to explain how the Universe has evolved.

3 Type Ia supernovae

3.1 Type Ia supernovae are thought to be standard candles and are bright enough to be observed out to red-shifts greater than z=1.0. Consequently they can be used to examine the expansion of the Universe around the current epoch.

3.2 Observations of remote type Ia supernovae show them to be fainter than expected. This has led to the conclusion that the expansion of the Universe is accelerating.

3.3 The ΛCDM model (Λ=cosmological constant, CDM=cold dark matter) has the current epoch dominated by matter and a cosmological constant. The Friedmann Equation for a flat Universe containing only matter and a cosmological constant is (after Ryden 2017):

$$H^2 = \left\{\frac{\dot{a}}{a}\right\}^2 = \frac{8\,\pi\,G}{3\,c^2}\,\varepsilon_{c,0}\left\{\frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0}\right\} \tag{5}$$

where H is the Hubble parameter; a the scale factor; $\varepsilon_{c,0}$ the current critical density; $\Omega_{m,0}$ the current density parameter for matter; $\Omega_{\Lambda,0}$ the current density parameter for a cosmological constant.

3.4 Observations of type Ia supernovae out to red-shifts a little beyond z=1.0 have shown that

$$\Omega_{m,0} \approx 0.3 \tag{6}$$

$$\Omega_{\Lambda,0} \approx 0.7 \tag{7}$$

3.5 This result is based solely on observations around the current epoch. It is independent of the early history of the Universe, in particular observations of the cosmic microwave background (CMB).

3.6 The dark matter hypothesis with a current density parameter, $\Omega_{d,0}$, of

$$\Omega_{d,0} \approx 0.25 \tag{8}$$

clearly accounts for the difference between equations (4) and (6). Matter is then made up of both the baryonic matter and the dark matter. This is the solution adopted by the ΛCDM model.

4 Early universe

4.1 It is generally accepted that the early Universe was dominated by radiation and matter alone, i.e. the cosmological constant played no part. The Friedmann Equation for a flat Universe containing only radiation and matter is (after Ryden 2017)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\,\pi\,G}{3\,c^2}\,\varepsilon_{c,0}\left\{\frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3}\right\} \tag{9}$$

where H is the Hubble parameter; a the scale factor; $\varepsilon_{c,0}$ the current critical energy density; $\Omega_{r,0}$ the current energy density of radiation relative to the critical energy density; $\Omega_{m,0}$ the current density parameter for matter.

4.2 For the current epoch, indicated by the 0 subscript

$$H_0^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\,\pi\,G}{3\,c^2}\,\varepsilon_{c,0} \tag{10}$$

4.3 Equation (9) can now be rewritten as

$$H_0\,dt = \frac{1}{\sqrt{\Omega_{r,0}}}\left(1 + \frac{a(t)}{a_{rm}}\right)^{1/2} a(t)\;da \tag{11}$$

where

$$\dot{a} = \frac{da}{dt} \tag{12}$$

and

$$a_{rm} = \frac{\varepsilon_{r,0}}{\epsilon_{m,0}} = \frac{\Omega_{r,0}}{\Omega_{m,0}} \tag{13}$$

a_{rm} is the scale factor when the energy densities for radiation and matter are equal; $\varepsilon_{r,0}$ is the current energy density of radiation; $\varepsilon_{m,0}$ the current energy density of matter.

4.4 Equation (11) can be integrated to give the time for any value of the scale factor

$$t = \frac{4\;a_{rm}^2}{3\,H_0\,\sqrt{\Omega_{r,0}}}\left[1 - \left(1 - \frac{a(t)}{2\,a_{rm}}\right)\left(1 + \frac{a(t)}{a_{rm}}\right)^{1/2}\right] \tag{14}$$

4.5 The horizon distance, D_h , at time t is defined as

$$D_h(t) = a(t) \int_0^t \frac{c}{a(t)} \, dt \qquad (15)$$

4.6 So equation (11) also leads to the horizon distance being given by

$$D_h(t) = \frac{2\, c\, a_{rm}\, a(t)}{H_0 \sqrt{\Omega_{r,0}}} \left[\left(1 + \frac{a(t)}{a_{rm}} \right)^{1/2} - 1 \right] \qquad (16)$$

4.7 At the time of last scattering the sound speed of the baryon-photon fluid, c_s , can be taken to be the same as that for a photon gas

$$c_s = \frac{c}{\sqrt{3}} \qquad (17)$$

leading to the sound horizon distance at last scattering, $S_h(t_{ls})$, being

$$S_h(t_{ls}) = \frac{D_h(t_{ls})}{\sqrt{3}} \qquad (18)$$

where $D_h(t_{ls})$ is the horizon distance at the time of last scattering, given by equation (15).

4.8 At the current epoch this has expanded to

$$S_h(t_0) = (1 + z_{ls})\, S_h(t_{ls}) \qquad (19)$$

4.9 The angular size (in degrees) on the sky now of the sound horizon at last scattering is

$$\theta_s = \frac{S_h(t_0)}{D_h(t_0)} \frac{180}{\pi} = \frac{D_h(t_{ls})}{D_h(t_0)} \frac{(1 + z_{ls})}{\sqrt{3}} \frac{180}{\pi} \qquad (20)$$

where $D_h(t_0)$ is the current horizon distance (~14,000 Mpc).

4.10 We also note the standard results that the temperature, T, is related to the scale factor by

$$\frac{T}{T_0} = \frac{a_0}{a} \tag{21}$$

and the red-shift, z, by

$$1 + z = \frac{1}{a} \tag{22}$$

4.11 We are now in a position that, given a scale factor, we can work out the red-shift, the temperature, the time, and the horizon distance.
Or given a temperature, we can work out the scale factor, the red-shift, the time, and the horizon distance.

4.12 We can address the consequences for cosmology of there being no dark matter. In terms of energy density, the Friedmann Equation is given by equation (1)

$$H^2 = \left\{\frac{\dot{a}}{a}\right\}^2 = \frac{8\,\pi\,G}{3\,c^2}\,\{\,\varepsilon_r + (\varepsilon_b + \varepsilon_d) + \varepsilon_\Lambda\} = \frac{8\,\pi\,G}{3\,c^2}\,\varepsilon_c \tag{23}$$

where ε_r is the energy density of radiation; ε_b the energy density of baryons; ε_d the energy density of dark matter; ε_Λ the energy density of a cosmological constant.

4.13 If there is no dark matter then

$$\varepsilon_d = 0 \tag{24}$$

and

$$\varepsilon_m = \varepsilon_b + \varepsilon_d = \varepsilon_b \tag{25}$$

where ε_m is the energy density of matter.

4.14 With no dark matter we are left with a shortfall in the energy density if this is to remain the critical energy density. We can compensate for this by adjusting the Friedmann Equation in (at least) two different ways

1) Model 1: introduce a multiplicative factor that affects all the components equally, and
2) Model 2: introduce a multiplicative factor that affects only the the baryonic matter.

We now consider these two models separately.

5 Model 1

5.1 There is no dark matter and the energy density is made up of radiation, baryonic matter, and a cosmological constant. We compensate for the lack of dark matter by introducing a multiplicative factor into the Friedmann Equation that affects all the components equally

$$H^2 = \left\{\frac{\dot{a}}{a}\right\}^2 = \frac{8\,\pi\,G}{3\,c^2}\,\gamma\,\{\,\varepsilon_r + \varepsilon_b + \varepsilon_\Lambda\} \quad (26)$$

where γ is the multiplicative factor.

5.2 This has no effect on Big Bang Nucleosynthesis (BBN) and equation (4) still holds

$$\Omega_{b,0} = \frac{\varepsilon_{b,0}}{\varepsilon_{c,0}} = 0.048 \quad (27)$$

5.3 For type 1a supernovae, equation (6) becomes

$$\Omega_{m,0} = \gamma\,\Omega_{b,0} \approx 0.3 \quad (28)$$

Hence from equation (27)

$$\gamma \approx 6 \quad (29)$$

And

$$\Omega^*_{\Lambda,0} = \gamma\,\Omega_{\Lambda,0} \quad (30)$$

where $\Omega^*_{\Lambda,0}$ is the new value of the density parameter for the cosmological constant. The cosmological constant is reduced by the factor γ from its previous value. However, this creates no new problems and everything continues to work as before.

5.4 For the early Universe, equation (13) leads to

$$a_{rm} = \frac{\varepsilon_{r,0}}{\epsilon_{b,0}} = \frac{\Omega_{r,0}}{\Omega_{b,0}} = 1.9\text{E-}3 \quad (3)$$

5.5 Equation (22) gives

$$z \approx 530 \quad (32)$$

and equation (21) gives

$$T \approx 1460K \quad (33)$$

This means radiation-matter equality takes place much later than the cosmic microwave background (CMB). The CMB now occurs when the Universe is still radiation dominated. This is different from the ΛCDM model where the CMB occurs when the Universe is matter dominated. These results are given in the 'Model 1' column of Table 7.2.

5.6 The CMB is formed at the time of so-called last scattering. This still occurs when the scale factor $a=9.18E-4$, at a red shift of $z \approx 1090$, and a temperature of $T \approx 2970K$. However, the CMB now happens at a time of around only 220,000y (rather than ~370,000y). The horizon distance is only ~0.14Mpc (rather than ~0.25Mpc). These results are given in the 'Model 1' column of Table 7.3.

5.7 The sound horizon is only 0.08Mpc (rather than 0.15Mpc) corresponding to an angular size on the sky of about 0.4 degrees. This is at odds with the first peak in the power spectrum of the CMB which occurs at an angular size of around 0.8 degrees. The observed first peak implies a wavelength at least double the sound horizon distance, which is physically impossible. With waves we can always work with the fundamental and overtones to get at smaller wavelengths, but we cannot get to anything larger than the fundamental wavelength. This is a knockout blow for Model 1 and effectively rules it out.

5.8 To summarise the ratio of radiation to matter in Model 1 causes the first peak in the CMB power spectrum to occur at around 0.4 degrees, which is completely different from the observed 0.8 degrees. This discrepancy is sufficient to rule out Model 1.

6 Model 2

6.1 There is no dark matter and the energy density is made up of radiation, baryonic matter, and a cosmological constant. We compensate for the lack of dark matter by introducing a multiplication factor into the Friedmann Equation that affects only the baryonic matter

$$H^2 = \left\{\frac{\dot{a}}{a}\right\}^2 = \frac{8\,\pi\,G}{3\,c^2}\left\{\varepsilon_r + \gamma_b\,\varepsilon_b + \varepsilon_\Lambda\right\} \tag{34}$$

where γ_b is the multiplicative factor.

6.2 This has no effect on Big Bang Nucleosynthesis (BBN) and equation (4) still holds

$$\Omega_{b,0} = \frac{\varepsilon_{b,0}}{\varepsilon_{c,0}} = 0.048 \tag{35}$$

6.3 For type 1a supernovae, equation (6) becomes

$$\Omega_{m,0} = \gamma_b\ \Omega_{b,0} \approx 0.3 \tag{36}$$

Hence from equation (35)

$$\gamma_b \approx 6 \tag{37}$$

This is exactly the same as for Model 1, so no changes for type Ia supernovae.

6.4 There is no change to the cosmological constant and equation (7) continues to hold.

6.5 For the early Universe, equation (13) leads to

$$a_{rm} = \frac{\varepsilon_{r,0}}{\gamma_b\ \epsilon_{b,0}} = \frac{\Omega_{r,0}}{\gamma_b\ \Omega_{b,0}} = \text{2.9E-4} \tag{38}$$

6.6 Equation (22) gives

$$z \approx 3450 \tag{39}$$

and equation (21) gives

$$T \approx 9400K \tag{40}$$

This is exactly the same as for the ΛCDM model and means radiation-matter equality occurs much earlier than the CMB. These results are given in column 'Model 2' of Table 7.2.

6.7 The CMB occurs when the scale factor a=9.18E-4, at a red shift of z≈1090, and a temperature of T≈2970K. The CMB happens at a time of around 370,000y, and the horizon distance is around ~0.25Mpc. These values are exactly the same as for the ΛCDM model.

6.8 As for the ΛCDM model the sound horizon is around 0.15Mpc corresponding to an angular size on the sky of about 0.7 degrees. This is in complete agreement with the first peak of the CMB power spectrum, which occurs at an angular size of around 0.8 degrees. These results are given in column 'Model 2' of Table 7.3.

6.9 In conclusion, our Model 2 matches the ΛCDM model, and explains all the cosmological observations.

6.10 One slight negative aspect is the fact that the γ factor is only applied to the baryonic matter. A simple way around this would be for the γ factor to be given by

$$\gamma = \gamma_o^{(1-3w)} \tag{41}$$

where w is the parameter in the equation of state

$$P = w\ \varepsilon \tag{42}$$

Then for radiation (w=1/3) we have

$$\gamma_r = \gamma_o^0 = 1 \tag{43}$$

And for matter (w=0) we have

$$\gamma_b = \gamma_o^1 = \gamma_o \tag{44}$$

This way a multiplicative factor is applied to both radiation and matter, but turns out to be 1.0 radiation.

6.11 We can summarise how our Model 2 provides an alternative to the ΛCDM model by the following two rules.

6.12 Rule 1: In all those equations where local physics applies we use the baryonic density as is

$$\rho_b \rightarrow \rho_b \tag{45}$$

$$\Omega_b \rightarrow \Omega_b \tag{46}$$

where the → symbol stands for "is replaced by".
For example, this applies to the baryon-to-photon ratio of big bang nucleosynthesis (BBN).

6.13 Rule 2: In all those equations where the Friedmann Equation is used or the matter density is needed we use the baryonic density multiplied by our γ_b factor

$$\boldsymbol{\rho_m \rightarrow \gamma_b\ \rho_b} \tag{47}$$

$$\boldsymbol{\Omega_m \rightarrow \gamma_b\ \Omega_b} \tag{48}$$

where the → symbol stands for "is replaced by".
For example, many of the equations in chapter 7 "Anisotropies in the Microwave Sky" (Weinberg, 2008) are based on the Friedmann Equation and should lead to the same results if equations (47) and (48) are employed.

6.14 All the physics equations that lie behind the peaks in the CMB power spectrum are effectively unchanged. We are replacing the additive dark matter terms with the multiplicative energy scale variation factor. The end results are all exactly the same. As a consequence we expect the CMB peaks to lie in exactly the same locations and have exactly the same relative heights.

6.15 The unanswered question, of course, is where does the γ_b factor come from and why should it have a value of around 6. JoKe22 (2019) examined the data in the SPARC catalogue of disk galaxies (Lelli et al, 2016) from the point of view that variations in the energy scale exist. That paper showed that energy scale variations can explain the rotation curves of disk galaxies and that the effective mass of the galaxies is as least 5 times the observed baryonic mass. So paper JoKe22 comes up with a factor of around 6 for how energy scale variations influence the effective gravitational acceleration of disk galaxies. All we are doing here is applying the same factor to the baryonic matter in the Friedmann Equation.

7 Tables

Table 7.1 Current epoch

Quantity	ΛCDM	Model 1	Model 2
Radiation			
$\varepsilon_{r,o}$	4.383E5 eV m^{-3}	4.383E5 eV m^{-3}	4.383E5 eV m^{-3}
$\Omega_{r,0}$	9.000E-5	9.000E-5	9.000E-5
γ_r	1.000	6.456	1.000
$\gamma_r \times \Omega_{r,0}$	9.000E-5	5.811E-4	9.000E-5
Baryons			
$\varepsilon_{b,0}$	2.338E8 eV m^{-3}	2.338E8 eV m^{-3}	2.338E8 eV m^{-3}
$\Omega_{b,0}$	0.048	0.048	0.048
γ_b	1.000	6.456	6.456
$\gamma_b \times \Omega_{b,0}$	0.048	0.310	0.310
Dark Matter			
$\varepsilon_{d,0}$	1.276E9 eV m^{-3}	0.000 eV m^{-3}	0.000 eV m^{-3}
$\Omega_{d,0}$	0.262	0.000	0.000
γ_d	1.000	1.000	1.000
$\gamma_d \times \Omega_{d,0}$	0.262	0.000	0.000
Total Matter			
$\varepsilon_{m,0}$	1.510E9 eV m^{-3}	2.338E8 eV m^{-3}	2.338E8 eV m^{-3}
$\Omega_{m,0}$	0.310	0.048	0.048
γ	1.000	6.456	6.456
$\gamma \times \Omega_{m,0}$	0.310	0.310	0.310
Cosmological Constant			
$\varepsilon_{\Lambda,0}$	3.360E9 eV m^{-3}	5.205E8 eV m^{-3}	3.360E9 eV m^{-3}
$\Omega_{\Lambda,0}$	0.690	0.689	0.690
γ	1.000	6.456	1.000
TOTAL			
$\Sigma\varepsilon_{\alpha,0} = \varepsilon_{c,0}$	4.870E9 eV m^{-3}	7.543E8 eV m^{-3}	4.870E9 eV m^{-3}
$\Sigma\gamma_\alpha \times \Omega_{\alpha,0} = \Omega_0$	1.000	1.000	1.000

Cosmological parameters for the current epoch. The values for the ΛCDM model are from Ryden (2017).

Table 7.2. Radiation-matter equality

Quantity	ΛCDM	Model 1	Model 2
a_{rm}	2.90E-4	1.87E-3	2.90E-4
z_{rm}	3447	533	3447
T_{rm}	9398 K	1456 K	9398 K
t_{rm}	1.57E+12 s 4.98E+4 y	2.58E+13 s 8.18E+5 y	1.57E+12 s 4.98E+4 y
$D_h(t_{rm})$	1.00E+21 m 3.24E-1 Mpc	1.64E+22 m 5.32E-1 Mpc	1.00E+21 m 3.24E-1 Mpc

Cosmological parameters for when the energy densities of radiation and matter were equal. The scale factor, a_{rm}, is given by equation (12); the redshift, z_{rm}, by equation (21); the temperature, T_{rm}, by equation (20); the time, t_{rm}, by equation (13); the horizon distance, $D_h(t_{rm})$, by equation (15).

Table 7.3. Last scattering

Quantity	ΛCDM	Model 1	Model 2
a_{ls}	9.18E-4	9.18E-4	9.18E-4
z_{ls}	1089	1089	1089
T_{ls}	2970	2970	2970
t_{ls}	1.17E+13 s 3.72E+5 y	6.91E+12 s 2.19E+5 y	1.17E+13 s 3.72E+5 y
$D_h(t_{ls})$	7.95E+21 m 2.58E-1 Mpc	4.29E+21 m 1.39E-1 Mpc	7.95E+21 m 2.58E-1 Mpc
$S_h(t_{ls})$	0.149 Mpc	0.080 Mpc	0.149 Mpc
θ_{ls}	0.664 deg	0.358 deg	0.664 deg

Cosmological parameters for the time of last scattering when the temperature, T_{ls}, was 2970K. The scale factor, a_{ls}, is given by equation (20); the redshift, z_{ls}, by equation (21); the time, t_{ls}, by equation (13); the horizon distance, $D_h(t_{ls})$, by equation (15); the sound horizon distance, $S_h(t_{ls})$, by equation (18); the angular size, θ_{ls}, by equation (19).

8 Discussion

8.1 In this paper we have shown that we can explain the main results of cosmology using the conjecture that the energy scale varies from location to location and that the effective baryonic energy density should be increased by a multiplicative factor of around 6. This means that there is no need to invoke the existence of dark matter, which acts as a separate additive component to baryonic matter.

8.2 A similar result has already been shown to explain the observed rotation curves of disk galaxies (JoKe22, 2019).

8.3 This paper rewrites the Friedmann Equation so that it depends on a multiplicative factor being applied to the baryonic matter. This replaces the dark matter term in the equation.

8.4 We need to be able to derive this new Friedmann Equation for a homogeneous isotropic universe, but one in which the energy scale can vary from location to location. A simple attempt at doing this was presented in JoKe18 (2018). What we need to do now is produce a rigorous derivation. This must be the subject of future work.

8.5 We can now summarise the cosmological implications of replacing dark matter with variations of the energy scale.

8.6 No change to the baryon-to-photon ratio.

8.7 No change to the interpretations of observations of type Ia supernovae

8.8 No change to the epoch of radiation-matter equality.

8.9 No change to the epoch of the cosmic microwave background (CMB).

8.10 No change to the angular size of the fluctuations in the CMB.

8.11 No change to the peaks in the CMB power spectrum.

8.12 We conclude by asserting that the major results of cosmology can be reproduced by replacing the hypothesis of dark matter with the conjecture of variation of the energy scale.

9 References

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