On the variation of the energy scale 2

Galaxy rotation curves

by

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Summary

This paper follows the original paper "On the variation of the energy scale" where Jo.Ke put forward the hypothesis that the flat rotation curves of spiral galaxies arise from variations in the energy scale. That paper assumed a point mass at the galaxy centre and the resultant rotation curves, although giving good fits to the outer regions, did not match the innermost few kiloparsecs.

This paper models a spiral galaxy as a disk with a Gaussian density distribution. When combined with the original Gaussian distribution for the energy scale variations more realistic galaxy rotation curves are obtained. The improved model is applied to the original six galaxies and the new fits are in much better agreement with the observed rotation curves.

1 Introduction

1.1 The paper "On the variation of the energy scale" (Jo.Ke, 2015) is referred to in this paper as simply "Jo.Ke 1".

1.2 The rotation curves of many spiral galaxies remain flat in their outer regions and do not show the fall off in speed expected if the majority of the mass is concentrated in the galaxy centre. The widely accepted explanation for these observations is that galaxies are embedded in large haloes of dark matter.

1.3 'Jo.Ke 1' put forward the hypothesis that the flat rotation curves are caused by variations in the energy scale. A simple Gaussian model for the variations gives good fits to the outer regions of spiral galaxies, i.e. beyond about 10kpc.

1.4 The assumption of a central point mass means that the rotation curves do not fit the inner few kiloparsecs at all.

1.5 This paper seeks to obtain a better fit for the inner parts of the rotation curve by adopting a more realistic mass distribution.

2 A note on the conservation of energy

2.1 In 'Jo.Ke 1' it was pointed out that energy is conserved at a local level but not globally. This needs a little clarification.

2.2 The ξ factor describing the energy scale variations is defined through the relation:

$$\xi_A \, E_{AX} = \xi_B \, E_{BX} = \xi_X \, E_{XX} \tag{1}$$

where ξ_A is the value of ξ at location A; E_{AX} is the value of the scalar energy at location X as measured by an observer at A; similarly for the other parameters.

2.3 It is clear that in general

$$E_{AX} \neq E_{BX} \neq E_{XX} \tag{2}$$

i.e. the values of the scalar energy at location X as measured by observers at A, B, X are not the same. Hence energy is not conserved.

2.4 However, it is obvious that $\xi_A \, E_{AX}$ does have the same value for all observers. So conservation of energy needs changing from E = const to ξE = const. In this sense we are back in control and conservation of energy still holds.

3 Galaxy model

3.1 'Jo.Ke 1' assumed a simple point mass for a galaxy. This was fine for the shape of rotation curves in the outer regions, but wrong for the inner regions.

3.2 We model a galaxy as a disk with an axisymmetric density distribution. This is illustrated in Figure 1.

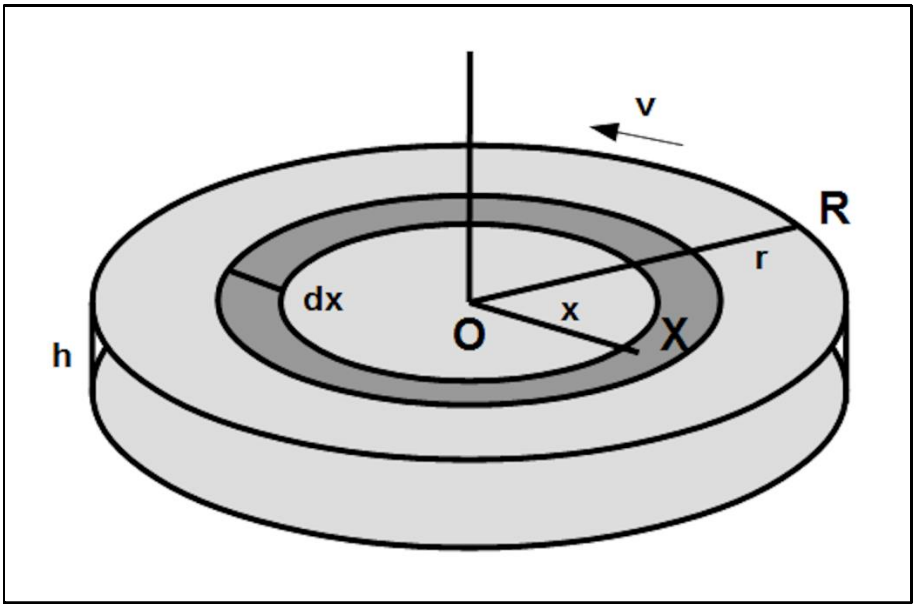

Figure 1: Model of a galaxy as a thin disk.

3.3 We assume a disk of thickness h, and a Gaussian density distribution given by:

$$\rho(r) = \rho_o \ exp(-r^2/\beta^2) \tag{3}$$

where ρ_o is the central density; β is 1/e-width of the density distribution; see Fig 2.

3.4 The mass, $dM(x)$, of an incremental ring, of width dx, is given by:

$$dM(x) = 2\,\pi\,x\,h\,\rho_o\,exp(-x^2/\beta^2)\,dx \tag{4}$$

3.5 The total mass of the galaxy is given by integrating equation (4) from 0 to infinity:

$$M = \pi\,\beta^2\,h\,\rho_o \tag{5}$$

3.6 The mass interior to location r is given by integrating equation (4) from 0 to r:

$$\boldsymbol{M(r) = M\,\{1 - exp(-r^2/\beta^2\,)\}} \qquad (6)$$

3.7 Equation (4) can be written as

$$\boldsymbol{dM(x) = M\,P(x)\,dx} \qquad (7)$$

where

$$\boldsymbol{P(x) = \frac{2x}{\beta^2}\,exp(-x^2/\beta^2\,)} \qquad (8)$$

This will be used later on.

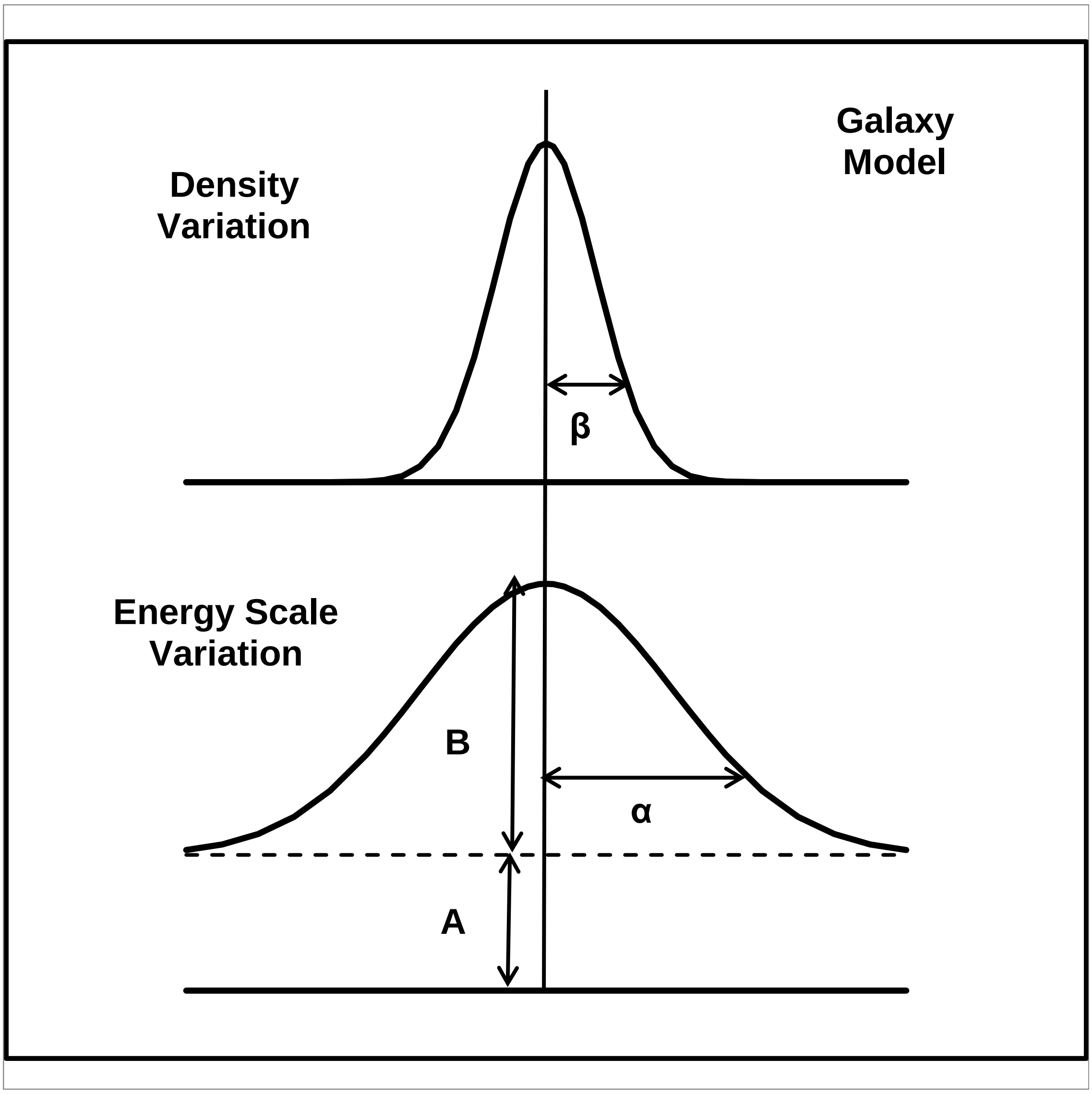

Figure 2: Galaxy Model comprising a narrow Gaussian density distribution and a broad Gaussian energy scale variation.

4 The rotation curves of galaxies

4.1 'Jo.Ke 1' suggested that the energy scale might vary from location to location. The dimensionless function of location, ξ, was introduced to describe the variations.

4.2 'Jo.Ke 1' assumed a simple Gaussian for the energy scale fluctuation:

$$\xi(r) = A + B \; exp(-r^2/\alpha^2) \tag{9}$$

where for any given fluctuation A and B are dimensionless constants; α is the 1/e-width of the fluctuation; see Figure 2.

4.3 'Jo.Ke 1' derived to the following expression for the rotation speed of stars round a central point mass:

$$v^2 = \frac{G\, M_{GG}}{r} \frac{\xi(0)}{\xi(r)} \tag{10}$$

where M_{GG} is the galaxy point mass as measured by an observer at the galaxy centre G; $\xi(0)$ is the value of the dimensionless function for the energy scale at G; $\xi(r)$ is the value of the function at location r.

4.4 We set

$$Q(r) = 1 + \gamma \, exp(-r^2/\alpha^2) \tag{11}$$

where

$$\gamma = B/A \tag{12}$$

4.5 Equation (10) can be written as

$$v^2 = \frac{G}{r} \frac{1}{Q(r)} \{M_{GG} Q(0)\} \tag{13}$$

4.6 We can now replace the term in parentheses (the galaxy point mass) with the exponential density distribution of equation (8)

$$v^2 = K^2 \frac{1}{Q(r)} \frac{\alpha}{r} \int_0^x Q(x) \; P(x) \, dx \tag{14}$$

where

$$K^2 = \frac{G\,M}{\alpha} \tag{15}$$

4.7 If the density profile is significantly narrower than the energy scale variation profile (i.e. $\beta < \alpha$), then $Q(x)$ can be replaced with $Q(0)$ and equation (14) simplifies to

$$v^2 = K^2 \; \frac{Q(0)}{Q(r)} \, \frac{M(r)}{M} \; \frac{\alpha}{r} \tag{16}$$

4.8 We set

$$S(r) = \sqrt{\frac{Q(0)}{Q(r)}} = \sqrt{\frac{1+\gamma}{1+\gamma\, exp(-r^2/\alpha^2)}} \tag{17}$$

and

$$T(r) = \sqrt{\frac{M(r)}{M}} = \sqrt{1 - exp(-r^2/\beta^2\,)} \tag{18}$$

4.9 Equation (16) becomes

$$v = K \; S(r) \; T(r) \, \sqrt{\alpha/r} \tag{19}$$

4.10 $S(r)$ is a pure number and depends on the two parameters α, γ. It describes the shape of the energy scale variation. It turns out that changes in $S(r)$ make a large difference to the outer part of the rotation curve, but little difference to the inner part.

4.11 $T(r)$ is also a pure number and depends on the single parameter β. It describes the shape of the mass distribution. It turns out that $T(r)$ is the opposite to $S(r)$ in that changes in $T(r)$ make a large difference to the inner part of the rotation curve, but little difference to the outer part.

4.12 If $S(r) = 1$, there is no energy scale variation and we are back with Keplerian orbits.
If we also have $T(r) = 1$, then we have Keplerian orbits for a central point mass.

4.13 The characteristics of equation (19) are illustrated in Fig 3 and Fig 4 below, where the normalisation simply assumes $K = 1.0$.

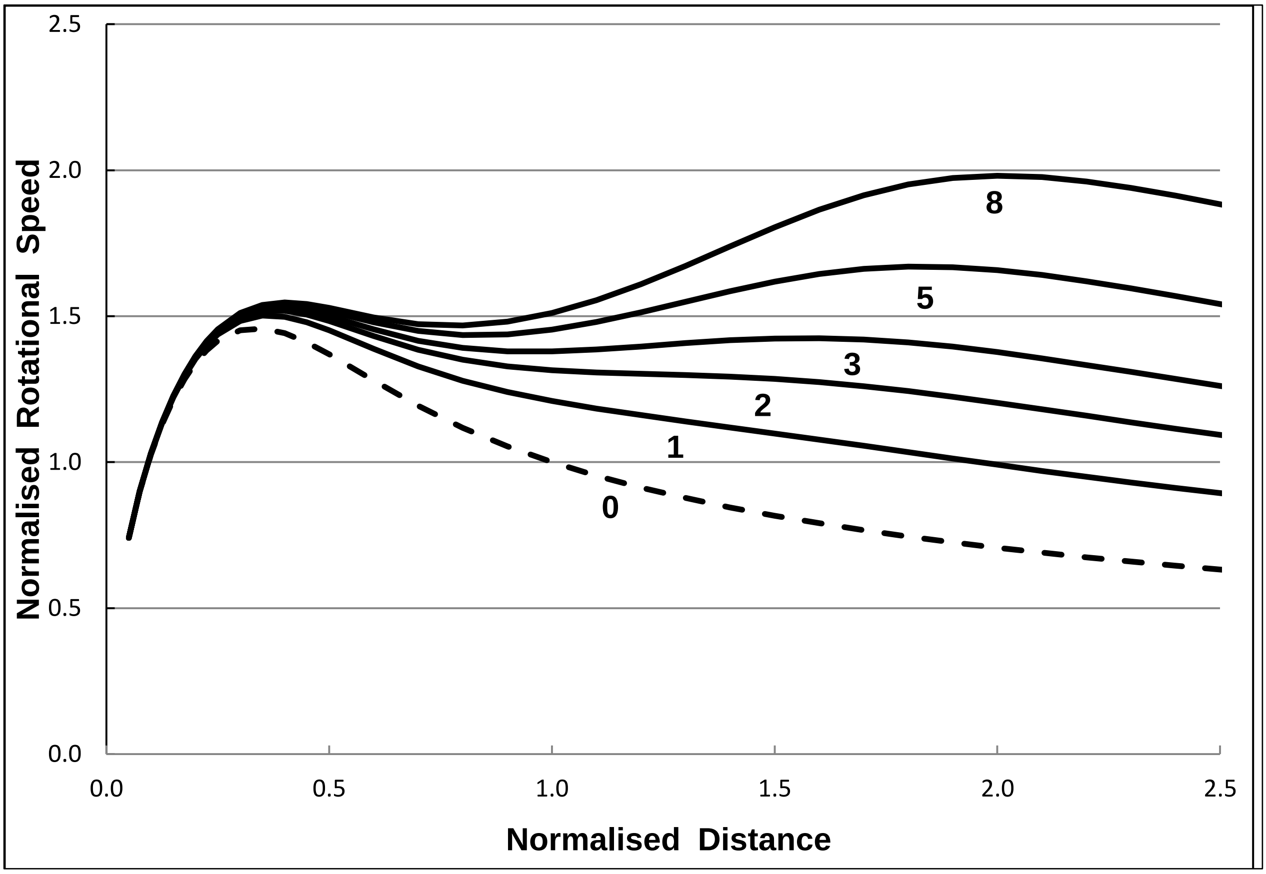

Fig 3: Galaxy rotation curves for different energy scale fluctuations

The rotation curves are for density scale value $\beta=0.3$ and for energy scale variation values $\gamma = 0, 1, 2, 3, 4, 5$. The curves are based on equation (19) with $K=1.0$. The curves show that variations in the energy scale produce large differences in the outer regions on the galaxy, but very small differences near the galaxy centre. The dashed curve ($\gamma=0$) is the curve for Keplerian orbits. The middle and dashed curves are the same as in Fig 2.

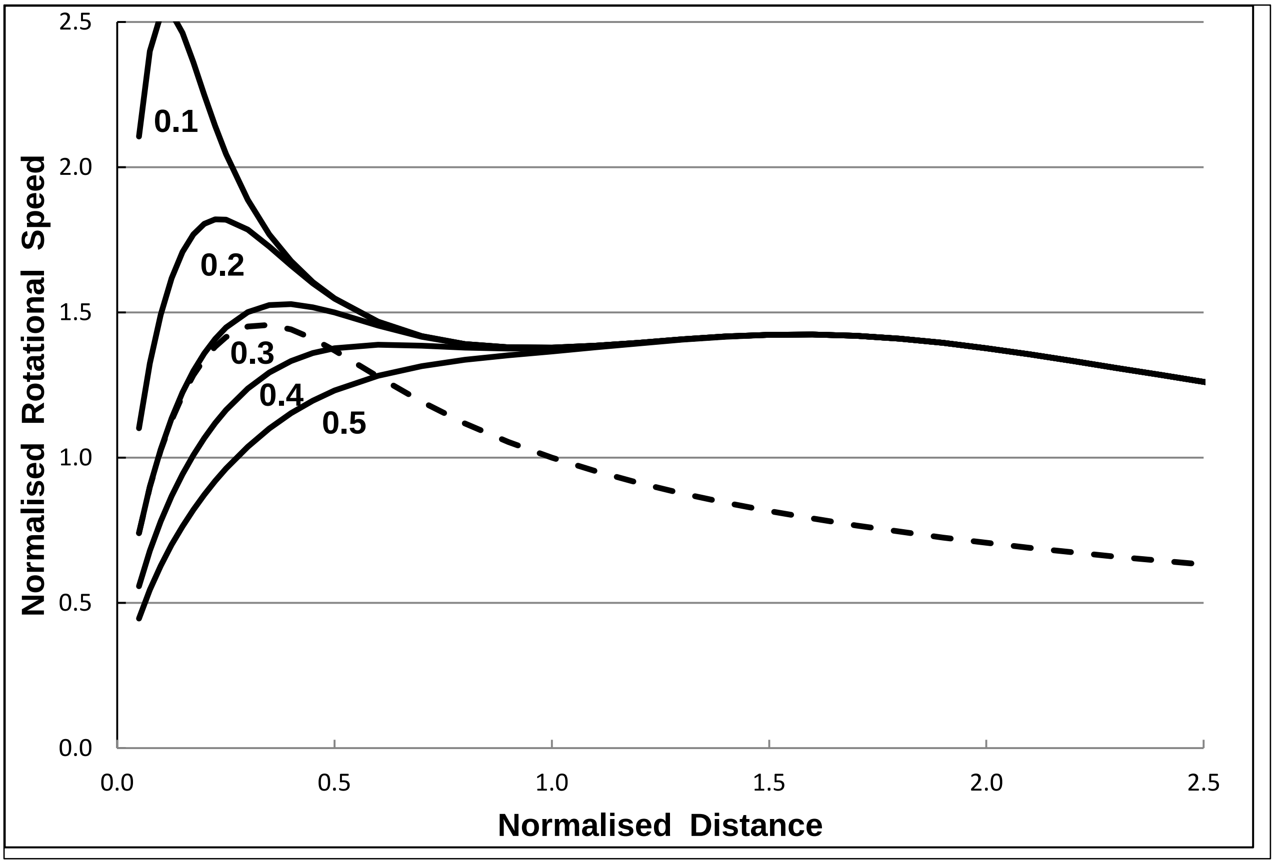

Fig 4. Galaxy rotation curves for different mass distributions.

The rotation curves for energy scale variation $\gamma=3.0$, and for density distributions $\beta= 0.1, 0.2, 0.3, 0.4, 0.5$. The curves are based on equation (19) with $K=1.0$. The curves show that variations in the density distribution produce large differences near the galaxy centre, but very small differences in the outer regions. The dashed curve ($\gamma=0$) is the curve for Keplerian orbits. The middle and dashed curves are the same as in Fig 1.

5 A sample of galaxy rotation curves

5.1 We work with the same sample of published galaxy rotation curves as in 'Jo.Ke 1' and see how well equation (19) fits. Equation (19) has four adjustable parameters ($\boldsymbol{\alpha}$, $\boldsymbol{\gamma}$, $\boldsymbol{K}$, $\boldsymbol{\beta}$) so some sort of fit is to be expected.

5.2 Equation (19) is used to give an approximate fit as it is somewhat easier to work with than equation (14). However, it assumes the density distribution is significantly narrower than the energy scale fluctuation ($\boldsymbol{\beta} < \boldsymbol{\alpha}$), which is the case for the spiral galaxies considered here.

5.3 Table (1) gives the values of the parameters in equation (19) as measured from the fits to the observed rotation curves.

5.4 The following figures show the rotation curves for six galaxies. The data points, shown as black diamonds, are the values taken from de Blok et al (2008). The solid line through the data points is an eye fit found by varying the free parameters in equation (19). The dashed line is the Keplerian curve for the same mass as for the eye fit.

5.5 In all the figures the eye fit curves revert to the expected $\boldsymbol{1/\sqrt{r}}$ fall off at large distances, but at a higher level than the Keplerian curves.

5.6 An estimate for the galaxy mass can be obtained from equation (16), when evaluated at the point $\boldsymbol{r = \alpha}$

$$\boldsymbol{M = \frac{v^2\,\alpha}{G}\left\{\frac{1+\ \gamma/e}{1+\gamma}\right\}\left\{\frac{1}{1-exp(-\alpha^2/\beta^2)}\right\}} \tag{20}$$

These values are also given in Table 1.

Table 1:

Galaxy	$\boldsymbol{\alpha}$ kpc	$\boldsymbol{\gamma}$	$\boldsymbol{K}$ km/s	$\boldsymbol{\beta}$ kpc	$\boldsymbol{v}$ km/s	$\boldsymbol{M}$ solar masses
NGC 2403	9	3.5	91	3.9	128	1.8×10^{10}
NGC 2841	15	2.0	225	5.0	296	1.8×10^{11}
NGC 2903	15	1.9	150	5.0	196	7.8×10^{10}
NGC 3198	20	2.4	113	7.9	152	6.0×10^{10}
NGC 3621	15	4.5	97	5.3	140	3.3×10^{10}
NGC 5055	20	2.2	149	7.0	198	1.0×10^{11}

Table 1. Parameters for equation (14) as derived from fitting the observed rotation curves for the listed galaxies. The rotation speed, $\boldsymbol{v}$, is measured at the point corresponding to the characteristic distance, $\boldsymbol{\alpha}$.

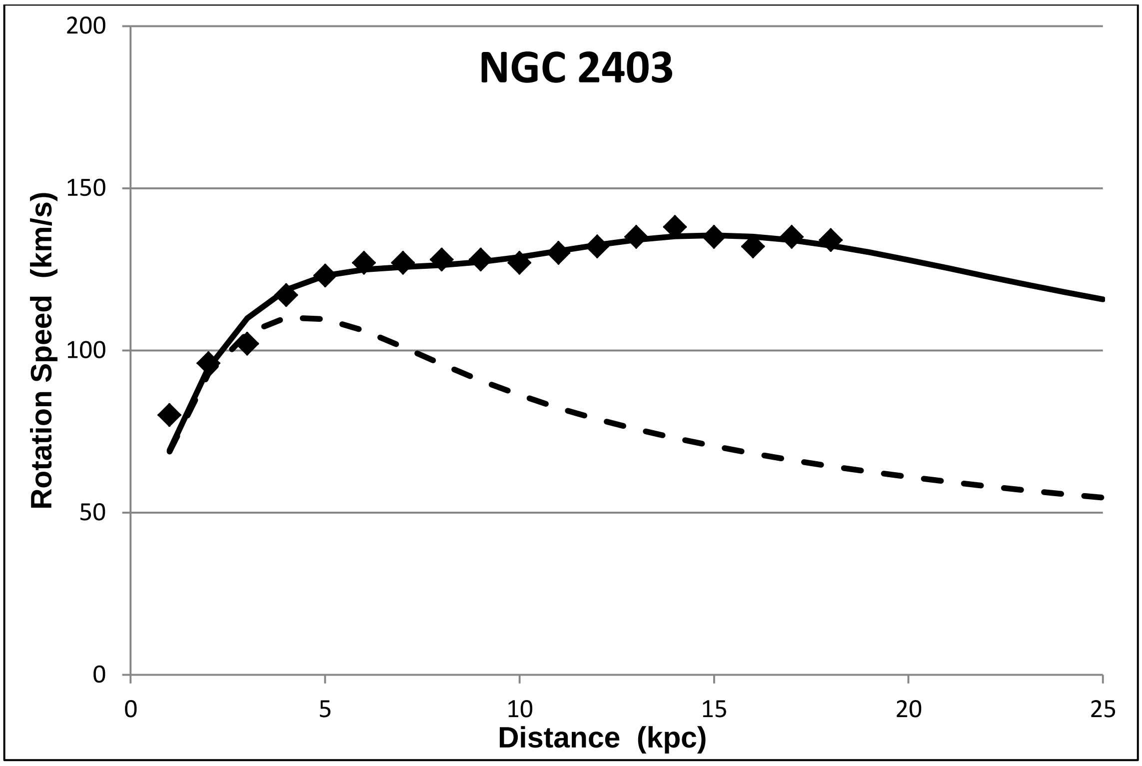

Figure 5: Rotation Curve for NGC 2403.

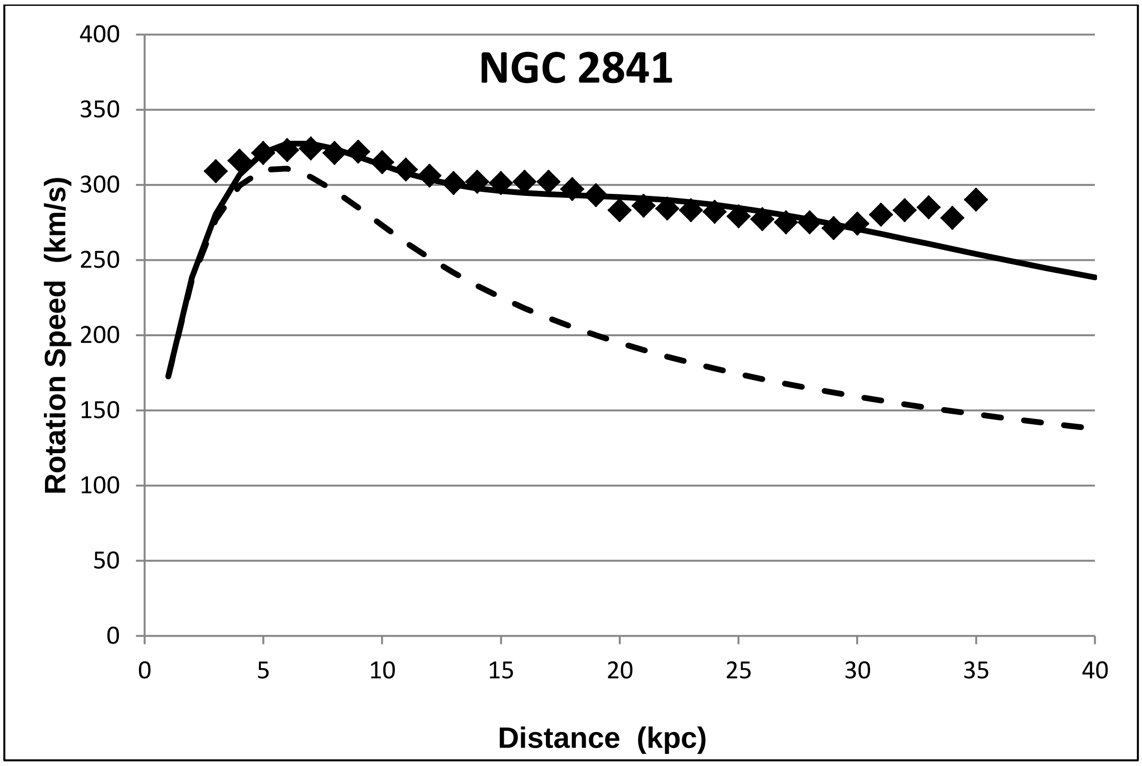

Figure 6: Rotation Curve for NGC 2481.

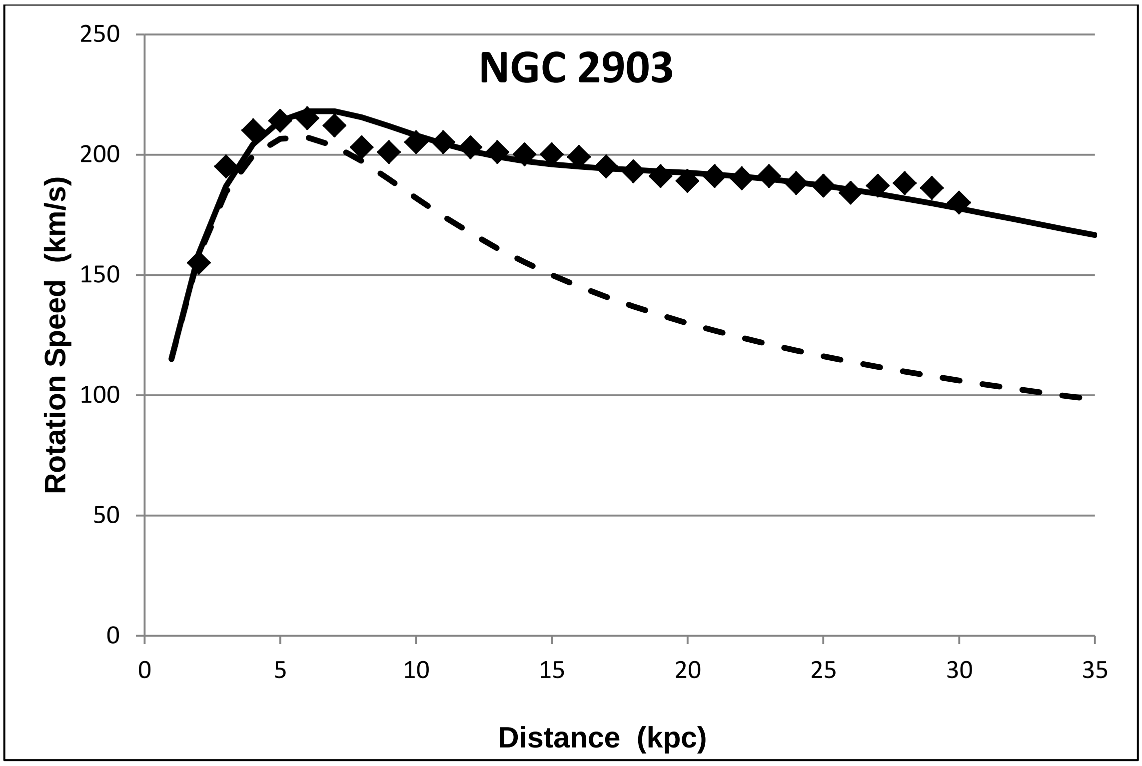

Figure 7: Rotation Curve for NGC 2903.

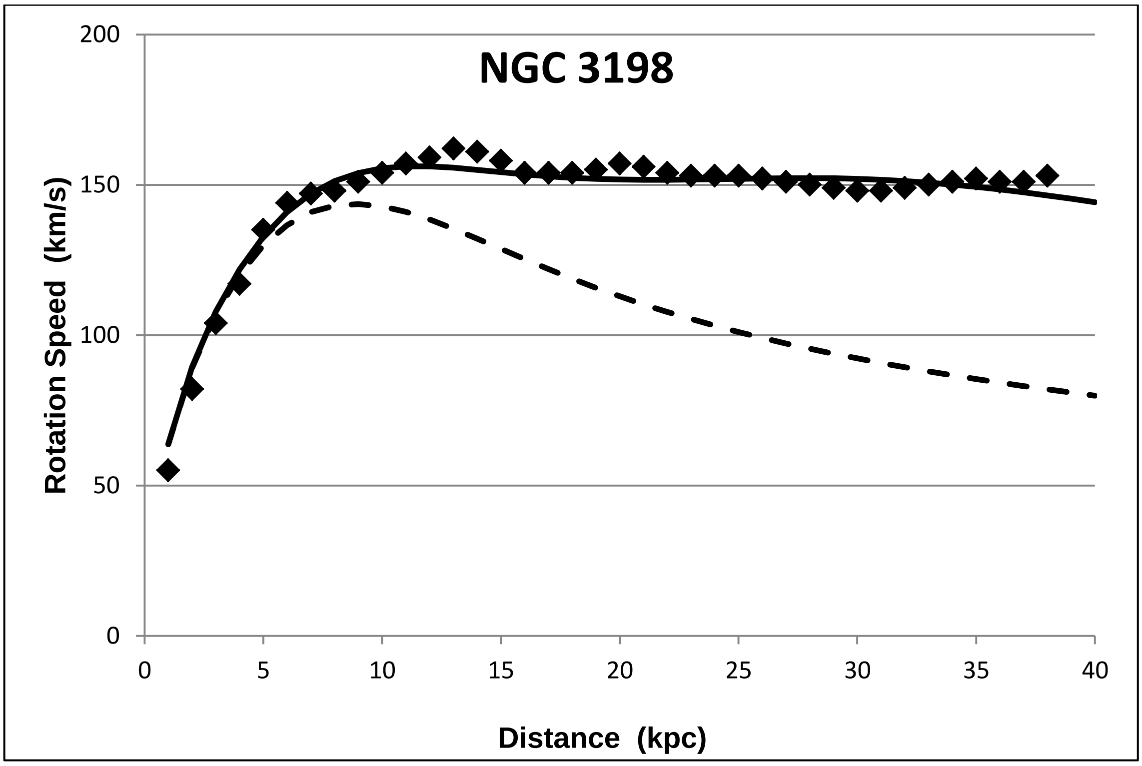

Figure 8: Rotation Curve for NGC 3198.

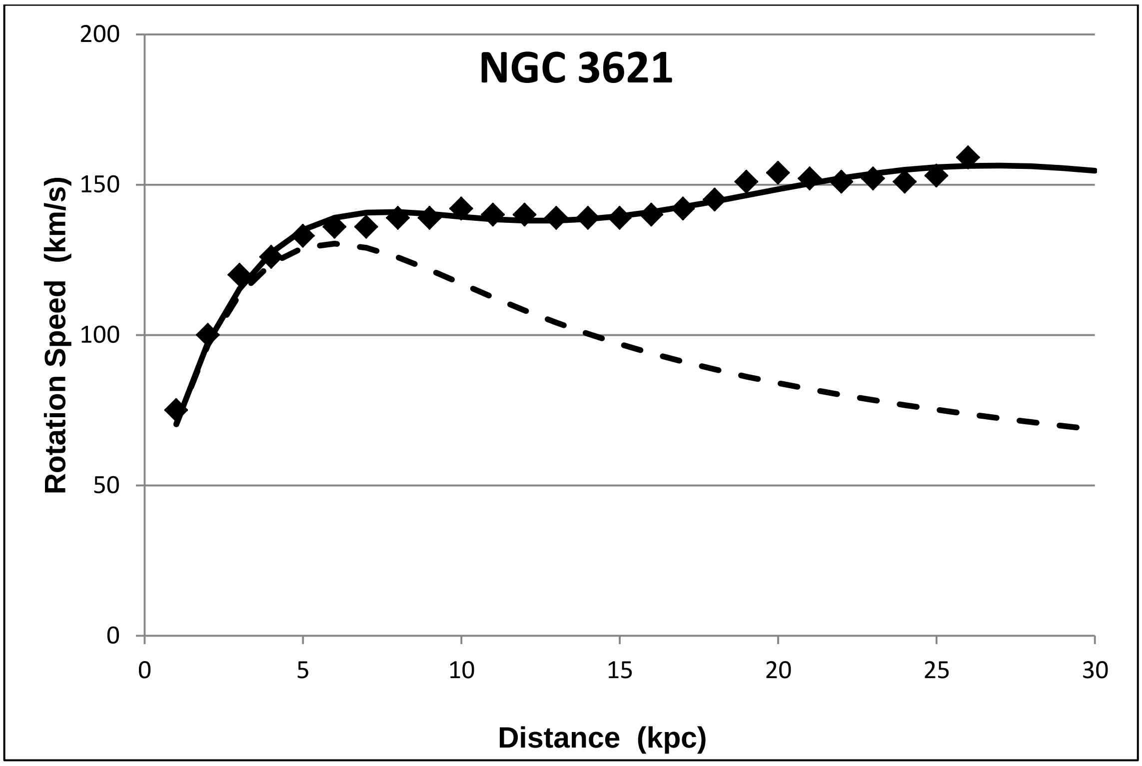

Figure 9: Rotation Curve for NGC 3621.

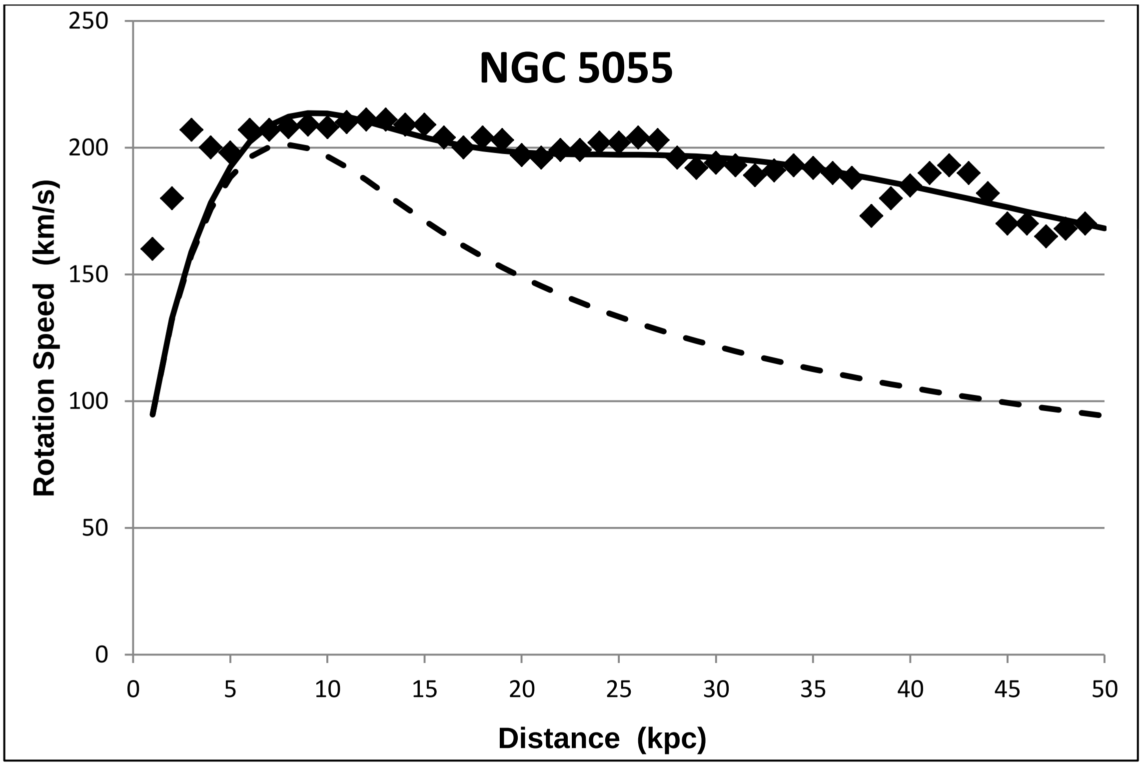

Figure 10: Rotation Curve for NGC 5055.

6 Comments

6.1 The new fits to the rotation curves show a big improvement in the galaxy centre regions in comparison to the curves displayed in 'Jo.Ke 1'.

6.2 The rotation curve fits are remarkably good considering the simple nature of the model, namely a simple Gaussian density distribution and a simple Gaussian energy scale fluctuation.

6.3 Of course, spiral galaxies are not smooth distributions of matter, but possess spiral arms and clumps of matter scattered across their disks. So it is not surprising that deviations from the fitted curves are in evidence.

6.4 The flat part of rotation curves (i.e. that part that deviates from Newtonian gravity) only becomes apparent beyond about 10kpc. The six galaxies examined here have all been chosen because they have rotation curve measurements beyond 10kpc.

6.5 From Table 1 it can be seen that

$$\boldsymbol{\alpha} \approx \ \mathbf{3}\,\boldsymbol{\beta} \tag{21}$$

so the simplifying assumption behind equation (19) holds. This also suggests a linkage between the shape of the mass distribution and the shape of the energy scale fluctuation. This is not altogether unexpected as the energy scale fluctuation must have some effect on how the galaxy evolves with time.

7 Conclusion

7.1 We extend the work put forward in 'Jo.Ke 1' that the observed flat rotation curves in spiral galaxies arise through changes in the energy scale from location to location.

7.2 No modifications have been made to Newton's law of gravitation. No dark matter has been introduced.

7.3 A simple Gaussian density distribution for the galaxy and a simple Gaussian for the fluctuations in the energy scale go a considerable way to reproducing the observed rotation curves.

7.4 We are not unaware that variations in the energy scale, if true, mean a paradigm shift for the whole of physics. But surely this is how it must be.

8 References

de Blok, WJG et al. (2008) The Astronomical Journal, **136**, 2648.
High-resolution rotation curves and galaxy mass models from THINGS.

Jo.Ke 1. (2015). "On the variation of the energy scale: an alternative to dark matter".