On the variation of the energy scale 19

The gravitational potential revisited

by

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Summary

Many physical theories are based on a scalar potential field, where the gradient gives rise to the force or acceleration. Separately, the hypothesis has been put forward that the energy scale can vary from location to location. Such energy scale variations have been used to explain the flat rotation curves of spiral galaxies and other astronomical observations, without requiring the existence of any dark matter. This paper looks at what can be said about the gravitational potential in the context of energy scale variations.

1 Introduction

1.1 JoKe1 (2015) puts forward the hypothesis that the energy scale can vary from location to location. It shows that the flat rotation curves of spiral galaxies can be explained by variations in the energy scale, without the need for any dark matter. JoKe2 (2015) introduces an improved model of a Gaussian distribution for the density and a Gaussian profile for the energy scale variation. JoKe3 (2015) applies this model of to a sample of 74 spiral galaxies, obtaining good fits in all cases.

1.2 JoKe11 (2017) looks at energy scale variations with a Gaussian profile to see whether any insights could be made into the nature of the gravitational potential. This paper extends the work of JoKe11 (2017) ...

2 Energy scale variations

2.1 Our hypothesis is that the energy scale can vary from location to location. This was first set out in JoKe1 (2015).

2.2 If we have an energy, $\boldsymbol{E}$, at location $\boldsymbol{X}$ and an observer at location $\boldsymbol{A}$ then our hypothesis means

$$\boldsymbol{E_{AX}\,\xi(A) = E_{XX}\,\xi(X)} \tag{1}$$

where $\boldsymbol{E_{AX}}$ is the energy at $\boldsymbol{X}$ as measured by an observer at $\boldsymbol{A}$; $\boldsymbol{E_{XX}}$ is the energy at $\boldsymbol{X}$ as measured by an observer at $\boldsymbol{X}$; $\boldsymbol{\xi(A)}$ is the value at $\boldsymbol{A}$ of the dimensionless function that describes the energy scale; $\boldsymbol{\xi(X)}$ is the value of the dimensionless function at $\boldsymbol{X}$.

2.3 For masses, using $\boldsymbol{E = m\,c^2}$, this becomes

$$\boldsymbol{\frac{\{M_{AX}\,c^2\,\xi(A)\}}{c^2} = \frac{\{M_{XX}\,c^2\,\xi(X)\}}{c^2}} \tag{2}$$

2.4 Our hypothesis is that only the energy scale that varies from location to location; i.e. the length scale does not vary and the time scale does not vary. Hence the speed of light remains an absolute constant and the equation for mass is

$$\boldsymbol{M_{AX} \;=\; M_{XX}\left\{\frac{\xi(X)}{\xi(A)}\right\}} \tag{3}$$

2.5 In most physical situations the object and the observer are at the same location. For example: particle physics experiments using the Large Hadron Collider. The one situation where they are in different locations is gravity. For our hypothesis Newtonian gravitational acceleration is given by

$$\boldsymbol{\ddot{r} \;=\; -\,\frac{G\,M_{OO}}{r^2}\left\{\frac{\xi(O)}{\xi(r)}\right\} \;=\; -\,\frac{G\,M}{r^2}\left\{\frac{\xi(O)}{\xi(r)}\right\}} \tag{4}$$

where $\boldsymbol{M_{OO} = M}$ is the mass of the object at $\boldsymbol{O}$ as measured by an observer at $\boldsymbol{O}$ (i.e. the 'intrinsic' mass); $\boldsymbol{r}$ is the distance; $\boldsymbol{\xi(O)}$ the value of the $\boldsymbol{\xi}$ function at $\boldsymbol{O}$; $\boldsymbol{\xi(r)}$ the value of the $\boldsymbol{\xi}$ function at $\boldsymbol{r}$.

2.6 Equation (4) shows clearly that we are modifying Newton's law of gravity. However, it still depends on the mass, and it still depends on the inverse square of the distance.

2.7 If, at small distances, the $\{\boldsymbol{\xi}(\boldsymbol{O})/\boldsymbol{\xi}(\boldsymbol{r})\}$ factor is close to 1 then our modified law reduces to Newton's law (as it must). So we should not expect there to be any detectable differences in the neighbourhood of the gravitating mass. For example: normal Newtonian gravitation should apply in the centres of all galaxies and there should be no requirement to invoke dark matter to explain any phenomena there.

2.8 If, at large distances, the $\{\boldsymbol{\xi}(\boldsymbol{O})/\boldsymbol{\xi}(\boldsymbol{r})\}$ factor is close to $\boldsymbol{\xi}(\boldsymbol{O})$ then again we have Newton's law with the mass $\boldsymbol{M}$ replaced by $\boldsymbol{M}\,\boldsymbol{\xi}(\boldsymbol{O})$. This means the mass can behave as if larger than its 'intrinsic' mass. This enables us to explain the rotation curves of spiral galaxies without invoking dark matter. We simply require the energy scale variation to make the central mass behave as larger than its intrinsic value.

2.9 Our hypothesis is different from allowing the gravitational constant, $\boldsymbol{G}$, to vary. In that case equation (4) can be written as

$$\ddot{\boldsymbol{r}} \;=\; -\;\left\{\frac{\boldsymbol{G}\;\boldsymbol{\xi}(\boldsymbol{O})}{\boldsymbol{\xi}(\boldsymbol{r})}\right\}\frac{\boldsymbol{M}}{\boldsymbol{r}^2} \;=\; -\boldsymbol{G}(\boldsymbol{r})\frac{\boldsymbol{M}}{\boldsymbol{r}^2} \tag{5}$$

where $\boldsymbol{G}(\boldsymbol{r})$ is now a varying gravitational constant. If $\boldsymbol{G}$ varies then this has serious implications in other areas of physics. For example: the hydrodynamic stability of stars; where the nature of different types of stars would be substantially different from what is observed. In our hypothesis we assume that $\boldsymbol{G}$ is an absolute constant and does not vary from location to location.

2.10 The choice of the $\boldsymbol{\xi}(\boldsymbol{r})$ function that has been adopted in previous papers is a simple Gaussian profile sitting on top of a fixed background:

$$\boldsymbol{\xi}(\boldsymbol{r}) = \boldsymbol{A} + \boldsymbol{B}\;\; \boldsymbol{exp}(-\boldsymbol{r}^2/\boldsymbol{\alpha}^2\,) \tag{6}$$

where (for a given energy scale variation) $\boldsymbol{A}, \boldsymbol{B}$ are numerical constants; $\boldsymbol{\alpha}$ is the 1/e-width. This is illustrated in Fig 1.

2.11 It is convenient to write equation (6) as:

$$\boldsymbol{\xi}(\boldsymbol{r}) = \boldsymbol{A}\,\{\boldsymbol{1} + \boldsymbol{\beta}\;\boldsymbol{exp}(-\boldsymbol{r}^2/\boldsymbol{\alpha}^2\,)\} \tag{7}$$

where

$$\boldsymbol{\beta} \;=\; \boldsymbol{B}/\boldsymbol{A} \tag{8}$$

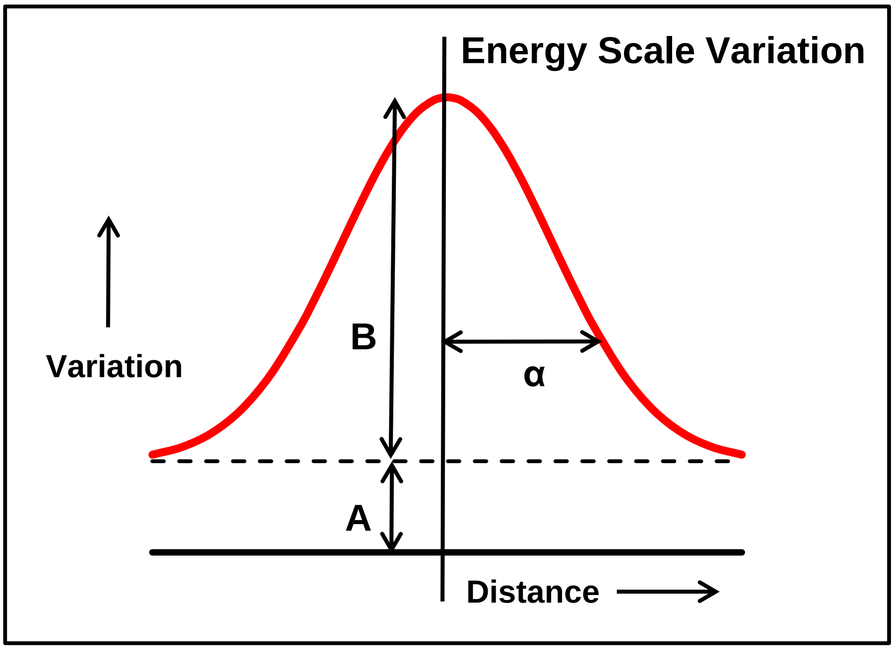

Figure 1. Schematic of an Energy Scale Variation as a Gaussian, height ***B***, 1/e-width ***α*** , sitting on top of of background with height ***A***.

2.12 At the origin, when $\boldsymbol{r = 0}$

$$\boldsymbol{\xi(0) = A\,\{1 + \beta\}} \tag{9}$$

2.13 The gravitational acceleration, equation (4), now becomes

$$\ddot{\boldsymbol{r}} \;=\; -\,\frac{\boldsymbol{G\,M}}{\boldsymbol{r^2}}\;\frac{\boldsymbol{(1 +\ \beta)}}{\boldsymbol{\{1 + \beta\ exp(-\,r^2/\alpha^2)\}}} \tag{10}$$

2.14 Near the mass, where $\boldsymbol{r \ll \alpha}$,

$$\boldsymbol{\xi(r) \approx A\,\{1 + \beta\}} \tag{11}$$

and equation (10) simplifies to

$$\ddot{\boldsymbol{r}} \;=\; -\,\frac{\boldsymbol{G\,M}}{\boldsymbol{r^2}} \tag{12}$$

This is normal Newtonian gravitation. As mentioned above, our hypothesis means no changes to Newtonian gravitation in the region close to the gravitating mass.

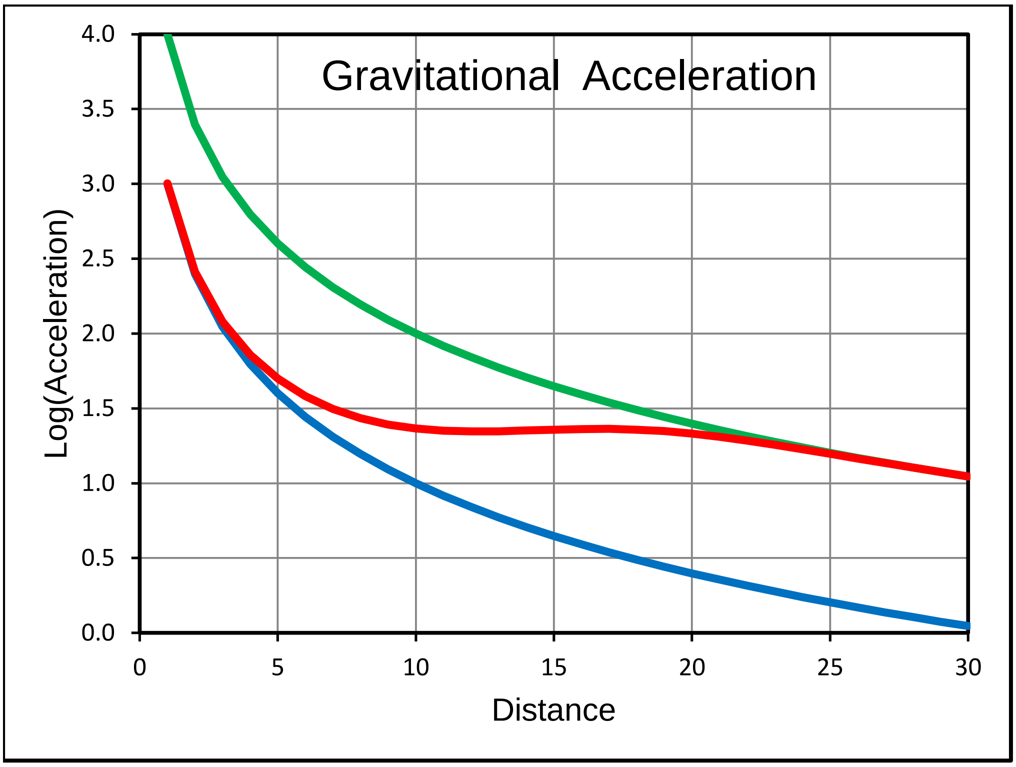

Figure 2. The variation of the gravitational acceleration with distance for a central point mass. The upper green curve shows the Newtonian gravitational acceleration for a high mass of M=10; the lower blue curve is the Newtonian gravitational acceleration for a low mass of M=1. The middle red curve is for an Energy Scale Variation that matches the green curve at large distances and matches the blue curve at small distances.

2.15 Far away from the mass, where $\boldsymbol{r} \gg \boldsymbol{\alpha}$,

$$\boldsymbol{\xi}(\boldsymbol{r}) = \boldsymbol{A}\ \{\mathbf{1} + \boldsymbol{\beta}\ \boldsymbol{exp}(-\boldsymbol{r}^2/\boldsymbol{\alpha}^2)\} \approx \ \boldsymbol{A} \tag{13}$$

and equation (10) simplifies to

$$\ddot{\boldsymbol{r}} = -\frac{\boldsymbol{G}}{\boldsymbol{r}^2}\,\boldsymbol{M}\,(\mathbf{1} + \boldsymbol{\beta}) \tag{14}$$

This is Newtonian gravitation with the mass behaving a factor $(\mathbf{1} + \boldsymbol{\beta})$ larger.

2.16 The behaviour of the gravitational acceleration is illustrated in Fig 2. The green curve is the relative acceleration for normal Newtonian gravity (equation 12) and a mass of 10. The blue curve is normal Newtonian gravity and a mass of 1. The red curve is the relative acceleration (equation 10) for an energy scale variation with α=10 and β=9.

2.17 So at very small distances and at very large distances the gravitational acceleration has an inverse-square law behaviour and behaves in the 'correct' Newtonian manner. It is only at intermediate distances that there is any departure from Newtonian gravitation. It is exactly in this zone where we observe the flat rotation curves of spiral galaxies.

2.18 Fig 2 shows that the energy scale variation acts as a 'interpolating function' between the two curves for normal Newtonian gravity. There must be a large number of functions that have an interpolating behaviour, not just the Gaussian form we have assumed in equation (6).

3 The gravitational potential

3.1 Our main focus in this paper is the gravitational potential simply because this is the usual starting point for most physical theories. And we would like to put Energy Scale Variations on a firmer theoretical basis.

3.2 Unfortunately we cannot integrate equation (10) analytically to give an expression for the gravitational potential everywhere. But we can generate approximate expressions for different regions.

3.3 At large distances: $\boldsymbol{r \gg \alpha}$, $\boldsymbol{exp(-r^2/\alpha^2) \approx 0}$, and the gravitational acceleration is given by equation (14). For spherical symmetry we have

$$-\nabla\varphi = -\frac{\partial\varphi}{\partial r} = -\frac{GM}{r}\ \{1+\beta\} \tag{15}$$

3.4 The gravitation potential is then given by

$$\varphi = \int_{\infty}^{r} \frac{GM}{r}\ \{1+\beta\}\ dr \tag{16}$$

which is integrated readily to give

$$\varphi = -\frac{GM}{r}\ \{1+\beta\} \tag{17}$$

assuming $\boldsymbol{\varphi}$ goes to zero at infinity.

3.5 This is the usual Newtonian gravitational potential, but with the central mass behaving as if it were a factor $\{\boldsymbol{1+\beta}\}$ larger. This is illustrated by the green curve in Figure 3.

3.7 At small distances: $\boldsymbol{r \ll \alpha}$, $\boldsymbol{exp(-r^2/\alpha^2) \approx 1}$, and the gravitational acceleration is given by equation (12). The integration is now

$$\varphi = \int_{R}^{r} \frac{GM}{r}\ dr + \int_{\infty}^{R} \frac{G\,M}{r^2} \left\{\frac{\xi(O)}{\xi(r)}\right\} dr \tag{18}$$

where $\boldsymbol{R}$ denotes the limit of the region where equation (12) holds.

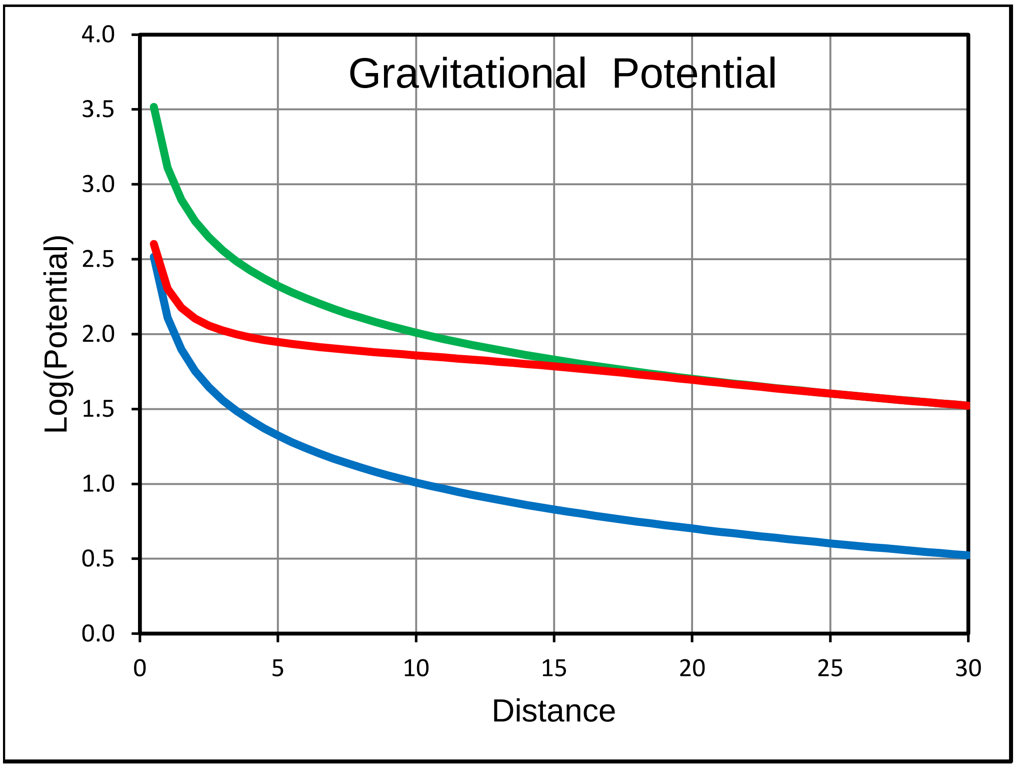

Figure 3. The variation of the gravitational potential with distance for a central point mass. The curves are the integration of the curves in Figure 2. The upper green curve is the Newtonian gravitational potential for a high mass of M=10; the lower blue curve is the Newtonian gravitational potential for a low mass of M=1. The middle red curve is for an Energy Scale Variation that matches the green curve at large distances and matches the blue curve at small distances.

3.8 Equation (18) is integrated readily to give

$$\boldsymbol{\varphi = -\frac{GM}{r} + constant} \tag{19}$$

Again this is the usual Newtonian gravitational potential with the central mass behaving normally. It is illustrated by the blue curve in Figure 3. However, we now have a constant of integration, which is not zero. This arises because the regime where equation (12) holds is for very small distances, so it is no longer valid to integrate in from infinity.

3.9 The constant in equation (19) is somewhat problematic. It means we can never approximate the gravitation potential at small distances to the Newtonian potential. In the interpolation zone we also do not have an algebraic expression for the gravitational potential. The only region where we have an acceptable expression for the gravitational potential is in the far field.

4.0 At small distances differences in the gravitational potential do approximate well to those for Newtonian gravity. But these differences are essentially the gravitational acceleration as defined by equation (10).

4.1 So our overall conclusion has to be that we have a good algebraic approximation for the gravitational acceleration but not for the gravitational potential. If theoretical considerations require the gravitation potential then we are not in a good position. However, if the theory only needs the gradient of the gravitational potential then we are in business.

4 General Relativity

4.1 Einstein's General Relativity is generally accepted to be our best theory of gravitation. So we need to examine whether the hypothesis of energy scale variations is compatible with it.

4.2 Many text books on General Relativity derive Einstein's field equations without any reference to the gravitational potential of the remote mass, $\boldsymbol{M}$. The field equations can be written as

$$\boldsymbol{G^{\lambda\mu} = k\ T^{\lambda\mu}} \tag{20}$$

where $\boldsymbol{G^{\lambda\mu}}$ is the Einstein tensor describing the curvature of space-time; $\boldsymbol{T^{\lambda\mu}}$ is the energy-momentum tensor; $\boldsymbol{k}$ is a constant to be determined. $\boldsymbol{G^{\lambda\mu}}$ in turn depends on the metric tensor, $\boldsymbol{g_{\lambda\mu}}$.

4.3 The spatial part of the geodesic equation in the Newtonian limit is

$$\boldsymbol{\frac{d^2x^\alpha}{dt^2} = -\frac{c^2}{2}\frac{\partial g_{00}}{\partial x^\alpha}} \tag{21}$$

where $\boldsymbol{x^\alpha}$ is a spatial coordinate; $\boldsymbol{g_{00}}$ is the 00 component of the metric tensor.

4.4 Equation (21) is compared to Newton's law of gravity

$$\boldsymbol{\frac{d^2x^\alpha}{dt^2} = -\frac{\partial\varphi}{\partial x^\alpha}} \tag{22}$$

where $\boldsymbol{\varphi}$ is the gravitational potential. This leads to the Newtonian limit of

$$\boldsymbol{g_{00} = 1 + \frac{2\,\varphi}{c^2} + \ ...} \tag{23}$$

since, at large distances, we have $\boldsymbol{\varphi \to 0}$ and $\boldsymbol{g_{00} \to 1}$.

4.5 Equation (23) is the first point at which the gravitational potential of a distant mass enters into General Relativity. By this stage all the really hard stuff of tensor calculus, curved manifolds, and the energy-momentum tensor has been been dealt with. The gravitational potential enters almost as an after-thought.

4.6 For large distances the Newtonian gravitation acceleration is given by

$$-\boldsymbol{\nabla\varphi} = -\frac{\partial\boldsymbol{\varphi}}{\partial\boldsymbol{r}} = -\frac{\boldsymbol{G\,M}}{\boldsymbol{r^2}} \tag{24}$$

This fixes the constant in equation (20) as

$$\boldsymbol{k} = \frac{\boldsymbol{8\,\pi\,G}}{\boldsymbol{c^4}} \tag{25}$$

4.7 For our hypothesis of energy scale variations we do not have a simple algebraic expression for the gravitational potential, $\boldsymbol{\varphi}$. However, equation (21) depends on the gradient of the potential, $\boldsymbol{\nabla\varphi}$, for which we do have an algebraic approximation

$$\frac{\partial\boldsymbol{\varphi}}{\partial\boldsymbol{r}} = -\frac{\boldsymbol{G\,M}}{\boldsymbol{r^2}}\,(\boldsymbol{1}+\boldsymbol{\beta}) \tag{26}$$

4.8 Hence, in the Newtonian limit of large distances, General Relativity should still hold but with the mass, $\boldsymbol{M}$, replaced by $\boldsymbol{M(1+\beta)}$. So our hypothesis of energy scale variations is compatible with General Relativity.

5 Discussion

5.1 Scalar-tensor theory

The function $\boldsymbol{\xi}(\boldsymbol{r})$ introduced in equation (4) describes the variation of the energy scale from location to location. It is a scalar function of position. In Einstein's theory of general relativity the gravitational field is described by the metric tensor and so, by definition, general relativity is a tensor theory. It can be argued that our introduction of a scalar function means we should be looking for a scalar-tensor theory of gravity, along the lines of the Brans-Dicke theory. However, we argue that we are not changing gravity, instead we are changing the energy scale and so we do not need to consider moving to a scalar-tensor theory. If some phenomenon, other that gravity, is found to depend on the energy scale then we would not have to introduce a new scalar field into that phenomenon. We would again argue that we are not changing the way that phenomenon works but simply changing the energy scale.

5.2 An action principle

What we would really like to do is formulate the hypothesis of energy scale variations in terms of an action principle and then apply the Principle of Least Action. This requires us to have an expression for the Lagrangian, which currently we do not have because we do not have a suitable expression for the gravitational potential. However, the Euler-Lagrange equations that come from applying the Principle of Least Action depend on the derivatives of the gravitation potential and not on the potential itself. This derivative is simply the gravitational force and we do have an (assumed) expression for this. Lagrangian mechanics and the Principle of Least Action for the rotation curves of spiral galaxies are discussed in paper JoKe18 (May 2018).

5.3 In conclusion: we do not have an analytical expression for the form of the gravitational potential under the regime of energy scale variations. We do have an expression for the gravitation acceleration that covers the low-velocity far-field regime.

8 References

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