On the variation of the energy scale 18

Rotation Curves and Lagrangian Mechanics

by

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Summary

Many physical theories are formulated in terms of Lagrangian mechanics and the Principle of Least Action. For the hypothesis that the energy scale can vary from location to location, it is shown that the rotation curves of spiral galaxies can be explained using Lagrangian mechanics. There is no need for any dark matter.

1 Introduction

1.1 JoKe1 (2015) puts forward the hypothesis that the energy scale can vary from location to location. It shows that the flat rotation curves of spiral galaxies can be explained by variations in the energy scale, without the need for any dark matter. JoKe2 (2015) introduces an improved model of a Gaussian distribution for the density and a Gaussian profile for the energy scale variation. JoKe3 (2015) applies this model of to a sample of 74 spiral galaxies, obtaining good fits in all cases.

1.2 One of the requirements for modern theories of physics is that they can be explained using Lagrangian mechanics; i.e. they possess a Lagrangian and can be derived from the Principle of Least Action.

1.3 This paper extends the work of previous papers in this series by formulating galaxy rotation curves in terms of Lagrangian mechanics. This puts the hypothesis of energy scale variations on a firmer theoretical basis.

2 Energy scale variations

2.1 Our hypothesis is that the energy scale can vary from location to location. This was first set out in JoKe1 (2015).

2.2 The variation in the energy scale is expressed through a scalar function of position, $\boldsymbol{\xi(r)}$.

2.3 The hypothesis leads to the Newtonian gravitational acceleration being given by

$$\ddot{\boldsymbol{r}} = -\frac{\boldsymbol{G\,M}}{\boldsymbol{r}^2}\left\{\frac{\boldsymbol{\xi(O)}}{\boldsymbol{\xi(r)}}\right\} \tag{1}$$

where $\boldsymbol{M}$ is the mass of the object at $\boldsymbol{O}$; $\boldsymbol{r}$ is the distance; $\boldsymbol{\xi(O)}$ is the value of the $\boldsymbol{\xi}$ function at $\boldsymbol{O}$; $\boldsymbol{\xi(r)}$ is the value of the $\boldsymbol{\xi}$ function at $\boldsymbol{r}$.

2.4 The choice made for the $\boldsymbol{\xi(r)}$ function in previous papers is that of a simple Gaussian profile sitting on top of a fixed background:

$$\boldsymbol{\xi(r) = A + B\ \ exp(-r^2/\alpha^2\,)} \tag{2}$$

where (for a given energy scale variation) $\boldsymbol{A, B}$ are numerical constants; $\boldsymbol{\alpha}$ is the 1/e-width. This is illustrated in Figure 1.

2.5 It is convenient to write equation (2) as:

$$\boldsymbol{\xi(r) = A\,\{1 + \beta\ exp(-r^2/\alpha^2\,)\}} \tag{3}$$

where

$$\boldsymbol{\beta\ =\ B/A} \tag{4}$$

2.6 At the origin, **O**, where $\boldsymbol{r = 0}$

$$\boldsymbol{\xi(0) = A\,\{1 + \beta\}} \tag{5}$$

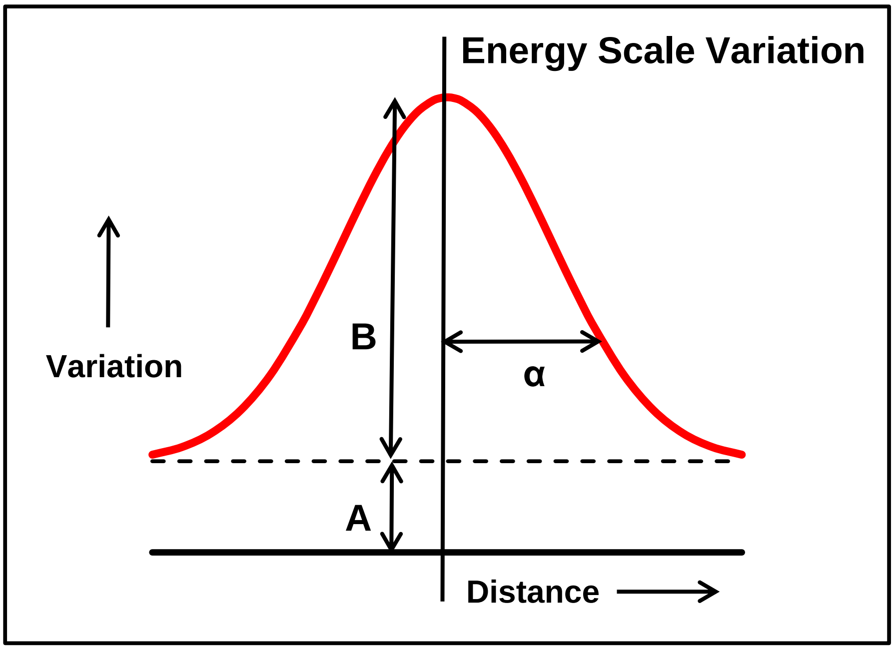

Figure 1. Schematic of an Energy Scale Variation as a Gaussian, height ***B***, 1/e-width ***α*** , sitting on top of of background with height ***A***.

2.7 The gravitational acceleration, equation (1), now becomes

$$\ddot{r} \;=\; -\,\frac{G\,M}{r^2}\,\frac{(1+\;\beta)}{\{1+\beta\;exp(-r^2/\alpha^2)\}} \tag{6}$$

2.8 The gravitational acceleration is, of course, the negative gradient of the gravitational potential, φ. So, in polar coordinates, we have

$$-\nabla\varphi = -\frac{\partial\varphi}{\partial r} = -\,\frac{G\,M}{r^2}\,\frac{(1+\;\beta)}{\{1+\beta\;exp(-r^2/\alpha^2)\}} \tag{7}$$

3 Lagrangian mechanics

3.1 We now look at how we can apply Lagrangian mechanics to galaxy rotation curves.

3.2 We consider a spiral galaxy to be a flat disk (a plane) and work in polar coordinates.

3.3 The Lagrangian, L, is

$$L = T - V \tag{8}$$

where T is the kinetic energy; V is the potential energy.

3.4 The Lagrange equation that has to be satisfied is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \tag{9}$$

where for polar coordinates: $i = 1, 2$; $q_1 = r$; $q_2 = \theta$; $\dot{q}_1 = \dot{r}$; $\dot{q}_2 = \dot{\theta}$.

3.5 The kinetic energy, T, for a small mass, m, is given by

$$T = \frac{m}{2}\left\{\dot{r}^2 + r^2\,\dot{\theta}^2\right\} \tag{10}$$

3.6 The gravitational potential energy, V, is given by

$$V = m\,\varphi(r) \tag{11}$$

where $\varphi(r)$ is the gravitational potential and a function of r only.

3.7 For the r component ($i=1$) of equation (9) we have using equation (10)

$$\frac{\partial L}{\partial \dot{q}_1} = \frac{\partial L}{\partial \dot{r}} = m\,\dot{r} \tag{12}$$

and

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) = m\,\ddot{r} = 0 \tag{13}$$

since for stable circular orbits we require $\ddot{r} = 0$

3.8 Next, for the second term in equation (9)

$$\frac{\partial L}{\partial q_1} = \frac{\partial L}{\partial r} = m\, r\, \dot{\theta}^2 - m\, \frac{\partial \varphi}{\partial r} \tag{14}$$

3.9 Substituting in from equation (7) we arrive at

$$r\, \dot{\theta}^2 = \frac{\partial \varphi}{\partial r} = \frac{G\, M}{r^2} \frac{(1 + \beta)}{\{1 + \beta\, exp(-r^2/\alpha^2)\}} \tag{15}$$

3.10 The rotational velocity, v, is simply

$$v = r\, \dot{\theta} \tag{16}$$

3.11 Finally we can write equation (15) as

$$v^2 = \frac{G\, M}{r} \frac{(1 + \beta)}{\{1 + \beta\, exp(-r^2/\alpha^2)\}} \tag{17}$$

3.12 We have used Lagrangian mechanics, and implicitly the Principle of Least Action, to derive the rotational velocity for a disk galaxy where the energy scale may vary from location to location.

3.13 Equation (17) is the equation used in papers JoKe1 (2015), JoKe2 (2015), JoKe3 (2015) to explain successfully the flat rotation curves of spiral galaxies. Every galaxy has its own values for the parameters α, β ; i.e. they are not universal constants.

7 Discussion

7.1 The hypothesis that the energy scale varies from location to location was first set out in JoKe1 (2015) and is used to explain the flat rotation curves of spiral galaxies without the need for any dark matter. Other papers in the series show that the hypothesis can explain all situations where dark matter is invoked, including: the high velocities of galaxies in clusters; collisions between clusters of galaxies; the growth of structure in the early Universe; the power spectrum of the cosmic microwave background; gravitational lensing.

7.2 An arbitrary algebraic expression has been assumed for the shape of the energy scale variation. This is a simple Gaussian superposed on a fixed background. This gives an algebraic expression for the gravitational acceleration, which eventually enables us to explain the rotation curves of spiral galaxies, without any dark matter.

7.3 Choosing an algebraic expression for the gravitational acceleration means we do not have an algebraic expression for the gravitational potential energy (we cannot do the integration). This ought to be a serious drawback to Lagrangian mechanics simply because the Lagrangian is constructed from the kinetic energy and the potential energy.

7.4 However, the application of the Principle of Least Action means it is the derivative of the gravitational potential that is required and not the potential itself. It is somewhat fortuitous that we have that derivative in the form of the gravitational acceleration and can proceed without further hindrance to the rotational velocity.

7.5 The use of Lagrangian mechanics puts the hypothesis of energy scale variations on a firmer theoretical footing. But, of course, it still does not prove that they exist.

8 References

JoKe1. (2015). "On the variation of the energy scale: An alternative to dark matter". www.varensca.com

JoKe2. (2015). "On the variation of the energy scale 2: Galaxy rotation curves". www.varensca.com

JoKe3. (2015). "On the variation of the energy scale 3: Parameters for galaxy rotation curves". www.varensca.com