On the variation of the energy scale 17

The Friedmann Equation and the Cosmic Microwave Background

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Summary

The Friedmann Equation is a key equation for describing how gravity affects cosmology. It includes contributions from the energy densities of radiation, baryonic matter, dark matter, and the cosmological constant. Separately, the hypothesis has been put forward that the energy scale can vary from location to location and that such energy scale variations can explain the rotation curves of spiral galaxies without the need for dark matter. This paper shows how the Friedmann Equation can be recast in terms of energy scale variations rather than dark matter.

Prior to recombination, energy scale variations would have formed gravitational wells. These would have given rise to baryon acoustic oscillations leading to the temperature fluctuations observed in the cosmic microwave background (CMB). The case for using the power spectrum of the CMB to support the existence of dark matter is now somewhat diminished because the same arguments can be used to support the existence of energy scale variations.

1 Introduction

1.1 JoKe1 (2015) put forward the hypothesis that the flat rotation curves of spiral galaxies are caused by variations in the energy scale. The first model took a point mass galaxy and a Gaussian for the energy scale variation. This gave good fits to the outer regions of spiral galaxies. The model was applied to a small sample of just six galaxies.

1.2 JoKe2 (2015) introduced an improved model for a spiral galaxy. This was made up of two components: (a) a narrow Gaussian density distribution, and (b) a broader Gaussian for the energy scale variation. As well as fitting the outer regions, this model also gave better fits to the inner regions. JoKe3 (2015) applied the model to a sample of 74 spiral galaxies and good fits were obtained in all cases.

1.3 JoKe8 (2016) looked at the primordial density perturbations and showed that these can be caused by energy scale variations in a similar manner to dark matter. JoKe12 (2017) looked at cosmology and showed how energy scale variations can replace dark matter in the Friedmann Equation. A single parameter characterising the energy scale variations is sufficient to give rise to Universes that are either decelerating, coasting, or accelerating.

1.4 This paper takes a fresh look at how the Friedmann Equation can be recast to work with energy scale variations in place of dark matter. Following this it is shown that energy scale variations can give rise to gravitational wells in a homogeneous medium and thence to baryonic acoustic oscillations as observed in the cosmic microwave background.

2 Energy scale variations

2.1 Previous papers in this series (e.g. JoKe1, 2015) put forward the hypothesis that the energy scale can vary from location to location.

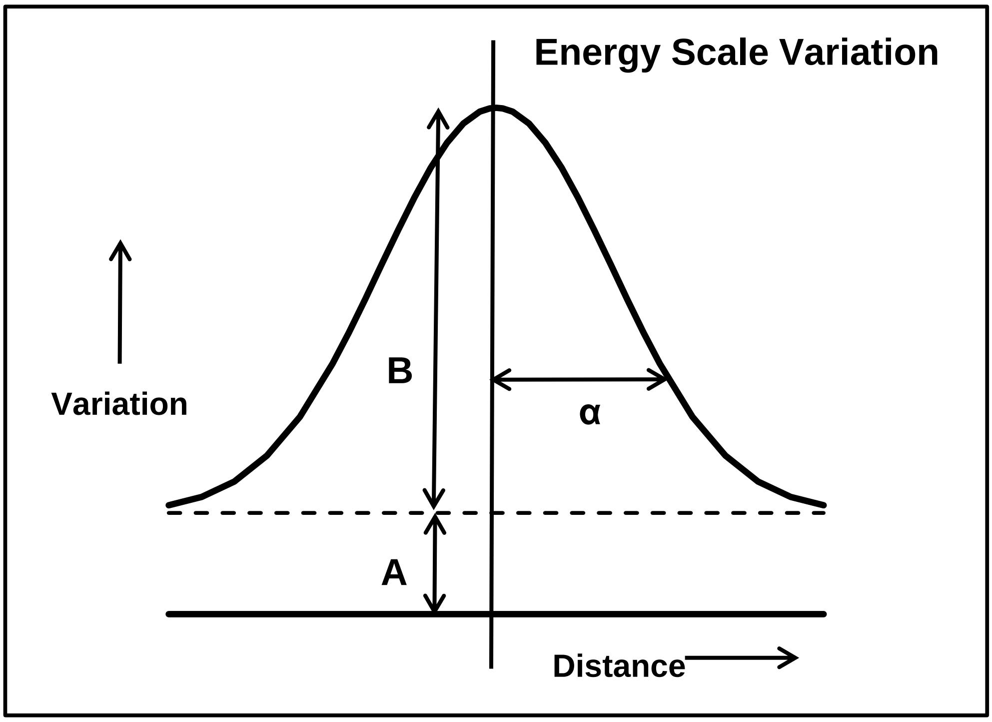

Fig 1. Schematic of an energy scale variation with a Gaussian profile. **A** is the base value; **B** is the height; **α** is the 1/e-width.

2.2 For a point mass, $\boldsymbol{m}$, the gravitational acceleration at $\boldsymbol{r}$ is given by

$$\ddot{\boldsymbol{r}} = -\frac{\boldsymbol{G\,m}}{\boldsymbol{r}^2}\left\{\frac{\boldsymbol{\xi(0)}}{\boldsymbol{\xi(r)}}\right\} \qquad (1)$$

where $\boldsymbol{\xi(r)}$ is the value of the energy scale variation at $\boldsymbol{r}$.
In essence this is normal Newtonian gravitation multiplied by the factor $\boldsymbol{\xi(0)/\xi(r)}$.

2.3 We assume a Gaussian energy scale variation (see Fig 1) given by

$$\boldsymbol{\xi(r) = A + B.exp(-r^2/\alpha^2)} \qquad (2)$$

where $\boldsymbol{A, B}$ are pure number constants; $\boldsymbol{\alpha}$ is a length constant.

2.4 And we set

$$\boldsymbol{\beta = B/A} \tag{3}$$

2.5 The gravitation acceleration, equation (1), is then

$$\ddot{r} = -\frac{G\,m}{r^2}\,\frac{\{1+\beta\}}{\{1+\beta\,exp(-r^2/\alpha^2)\}} \tag{4}$$

2.6 We want to move away from a point mass and consider instead a homogeneous sphere, radius $\boldsymbol{s}$, density $\boldsymbol{\rho}$. The gravitational acceleration is now given by an integration over the spherical shells of the homogeneous sphere. The mass of a thin shell is simply

$$dm = 4\,\pi\,x^2\rho\,dx \tag{5}$$

2.7 Equation (4) is then replaced by

$$\ddot{r} = \frac{-4\,\pi\,G\,\rho}{r^2\{1+\beta\,exp(-r^2/\alpha^2)\}}\int_0^s x^2\,\{1+\beta\,exp(-x^2/\alpha^2)\}\,dx \tag{6}$$

2.8 For large distances $\boldsymbol{r \gg \alpha}$

$$\frac{1}{r^2\{1+\beta\,exp(-r^2/\alpha^2)\}} \approx \frac{1}{r^2} \tag{7}$$

2.9 And for a large energy scale variation $\boldsymbol{s \gg \alpha}$ the second term of the integrand in equation (6) becomes

$$\int_0^s x^2\;exp(-x^2/\alpha^2)\,dx \approx \int_0^\infty x^2\;exp(-x^2/\alpha^2)\,dx = \frac{\sqrt{\pi}\,\alpha^3}{4} \tag{8}$$

2.10 Hence for $\boldsymbol{r \gg \alpha}$ and $\boldsymbol{s \gg \alpha}$ equation (6) is integrated to give

$$\ddot{r} = -\frac{4\,\pi\,G\,\rho}{r^2}\left\{\frac{s^3}{3} + \frac{\sqrt{\pi}\,\alpha^3\beta}{4}\right\} \tag{9}$$

2.11 The mass of the homogeneous sphere is

$$\boldsymbol{m} = \frac{\boldsymbol{4}}{\boldsymbol{3}} \boldsymbol{\pi} \, \boldsymbol{s}^3 \boldsymbol{\rho} \tag{10}$$

2.12 Equation (9) can be written as

$$\ddot{\boldsymbol{r}} = -\frac{\boldsymbol{G}\,\boldsymbol{m}}{\boldsymbol{r}^2}\left\{\boldsymbol{1} + \frac{\boldsymbol{3}\sqrt{\boldsymbol{\pi}}}{\boldsymbol{4}}\left(\frac{\boldsymbol{\alpha}}{\boldsymbol{s}}\right)^{3}\boldsymbol{\beta}\right\} \tag{11}$$

2.13 Both terms inside the braces are independent of $\boldsymbol{r}$, so the gravitational potential arising from the homogeneous sphere is

$$\frac{\boldsymbol{G}\,\boldsymbol{m}}{\boldsymbol{r}}\left\{\boldsymbol{1} + \frac{\boldsymbol{3}\sqrt{\boldsymbol{\pi}}}{\boldsymbol{4}}\left(\frac{\boldsymbol{\alpha}}{\boldsymbol{s}}\right)^{3}\boldsymbol{\beta}\right\} \tag{12}$$

2.14 For simplicity we set

$$\boldsymbol{\gamma} = \frac{\boldsymbol{3}\sqrt{\boldsymbol{\pi}}}{\boldsymbol{4}}\left(\frac{\boldsymbol{\alpha}}{\boldsymbol{s}}\right)^{3}\boldsymbol{\beta} \tag{13}$$

2.15 The gravitational potential can then be written as

$$\frac{\boldsymbol{G}\,\boldsymbol{m}}{\boldsymbol{r}}\{\boldsymbol{1} + \boldsymbol{\gamma}\} \tag{14}$$

This is essentially the usual Newtonian potential multiplied by a factor of $\{\boldsymbol{1} + \boldsymbol{\gamma}\}$.

2.16 Equation (14) is used later on when we modify the standard Friedmann Equation for a homogeneous sphere filled with a number of small energy scale variations.

3 The Friedmann Equation

3.1 The Friedmann Equation governs the expansion of a Universe that is both homogeneous and isotropic. It was derived by Alexander Friedmann in 1922 from Einstein's equations for general relativity.

3.2 Following Ryden (2017) the Friedmann Equation for a flat (zero curvature) matter-only (no radiation; no cosmological constant) is given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{4\,\pi\,G}{3}\{\rho_b + \rho_{DM}\} \tag{15}$$

where a is the usual scale factor; the dot indicates differentiation with respect to time; ρ_b the density of baryonic matter; ρ_{DM} the density of dark matter.

3.3 Ryden (2017) gives a simplified (non-relativistic) derivation of this based on a homogeneous sphere, Fig 2. This is paraphrased as follows:

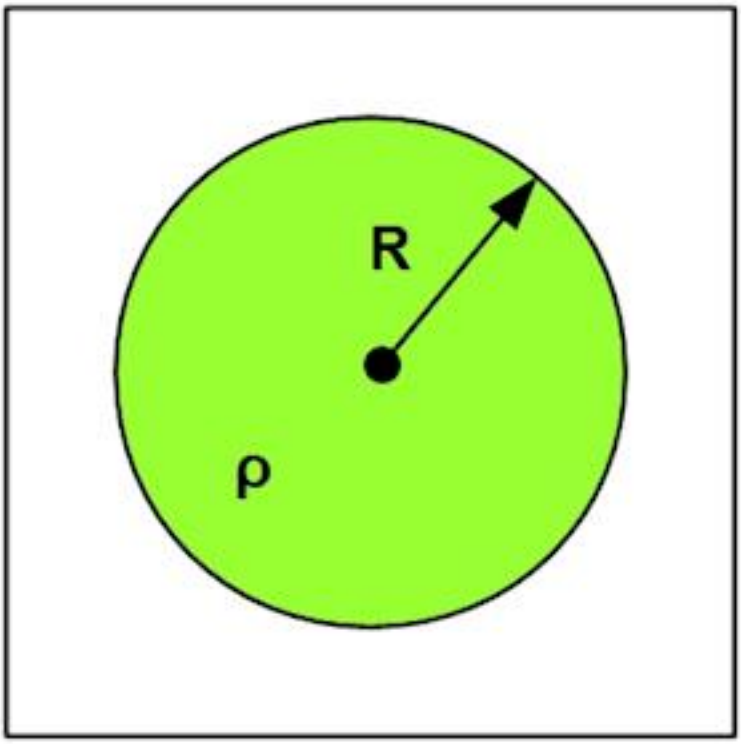

Fig 2. Homogeneous sphere; radius **R**; uniform density **ρ.**

3.4 Equating the kinetic energy (from the radial velocity) to the gravitational potential energy gives

$$\frac{1}{2}\left(\frac{\dot{R}}{R}\right)^2 = \frac{G\,M}{R} \tag{16}$$

where R is the radius; M the mass.

3.5 The mass of the sphere is given by

$$M = \frac{4}{3}\,\pi\,R^3\,\rho \tag{17}$$

where ρ is the uniform density.

3.6 For isotropic expansion or contraction the radius R can be written as

$$R = a\,r \tag{18}$$

where a is the scale factor; r the comoving radius (a constant).

3.7 Differentiating with respect to time gives

$$\dot{R} = \dot{a}\,r \tag{19}$$

3.8 Equations 16, 17, 18 & 19 are readily combined to give

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\,\pi\,G}{3}\,\rho \tag{20}$$

where $\rho = \rho_b + \rho_{DM}$

3.9 The Benchmark Model of Ryden (2017) has $\rho_{DM} = 5.46\,\rho_b$. So the Friedmann Equation (equation 20) for a Universe containing only baryonic matter and dark matter can be written as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\,\pi\,G}{3}\,(1+\beta)\,\rho_b \tag{21}$$

where $\beta = 5.46$.

3.10 We now derive the Friedmann Equation for a homogeneous sphere containing energy scale variations, as shown schematically in Fig 3.

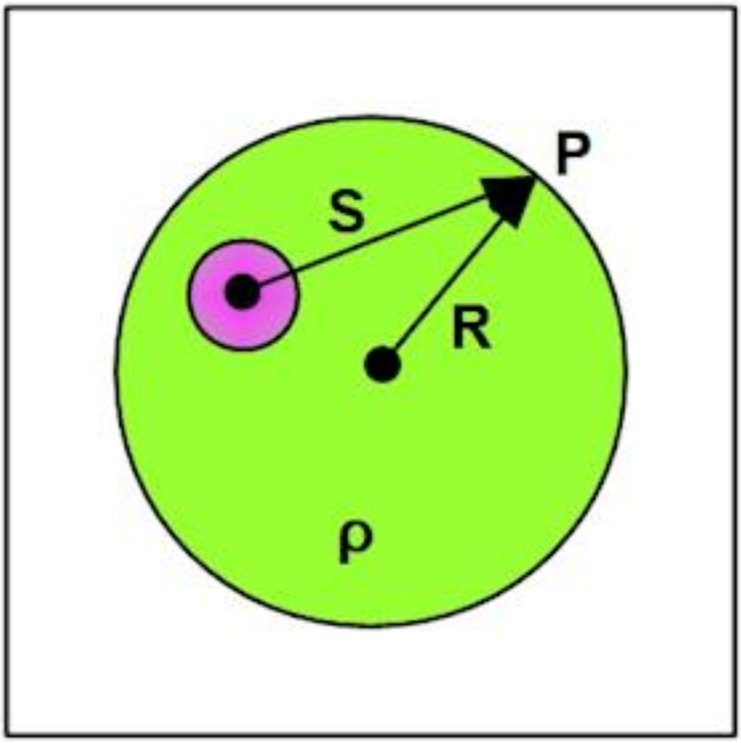

Fig 3. Homogeneous sphere: radius **R**; uniform density **ρ**. The purple disk represents an energy scale variation: size **D**; a distance **S** from surface point **P**.

3.11 We consider a small energy scale variation: radius **D**; height **β**; distance **S** from point **P** on the surface.

3.12 The gravitational potential at **P** caused by the energy scale variation (derived earlier) is given by equation (14)

$$\frac{G\,m}{S}\,\{1 + \gamma\} \tag{22}$$

where the mass of the energy scale variation is given by

$$m = \frac{4}{3}\,\pi\,D^3\,\rho \tag{23}$$

3.13 The addition to the gravitational potential over that already included in the Friedmann Equation is

$$\frac{G\,m}{S}\,\gamma \tag{24}$$

3.14 Working with comoving coordinates we can write

$$S = a\,s \tag{25}$$

$$D = a\,d \tag{26}$$

The additional gravitational potential is

$$\frac{4\,\pi\,G\,\rho}{3}\left(\frac{d^3}{s}\right)a^2\,\gamma \tag{27}$$

3.15 This leads to a modified Friedmann Equation of

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\,\pi\,G}{3}\left\{1 + \left(\frac{d^3}{r^2\,s}\right)\gamma\right\}\rho \tag{28}$$

3.16 We now consider the sphere (and the whole Universe) to be filled with, not just one energy scale variation, but a whole distribution. Note the sphere is still homogeneous; the density still has the constant value ρ. Equation (28) becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\,\pi\,G}{3}\left\{1 + \sum_i \frac{d_i^3\,\gamma_i}{r_i^2\,s_i}\right\}\rho \tag{29}$$

3.17 We set

$$\beta_{ESV} = \sum_i \frac{d_i^3\,\gamma_i}{r_i^2\,s_i} \tag{30}$$

to give

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\,\pi\,G}{3}\left(1 + \beta_{ESV}\right)\rho \tag{31}$$

3.18 Comparing equations (21) and (31) we see that we require $\boldsymbol{\beta_{ESV} = 5.46}$ to reproduce the 'observed' (derived) Friedmann Equation for a matter dominated flat Universe. This means there is no need to invoke the existence of dark matter; energy scale variations do the job just as well.

3.19 From JoKe3 (2015) a value of $\boldsymbol{\beta}$ derived from a set of 74 spiral galaxies is $\boldsymbol{\beta = 3.6 \pm 1.9}$; within the same ball-park as the 5.46 needed here.

3.20 The above puts on a firmer footing the work presented in JoKe12 (2017). There it was shown that the Friedmann Equation, as modified for energy scale variations, can lead to matter-only Universes that are decelerating, coasting, or accelerating. There is no need for any dark matter (or a cosmological constant).

5 Energy Scale Variation in a Homogeneous Medium

5.1 It is usually understood that the early Universe had a near uniform density (of baryons) right up to the time of recombination and the epoch of the cosmic microwave background. It is also usually assumed that dark matter started forming structures & potential wells much earlier on as it was not subject to photon & electron scattering. We now look at how an energy scale variation in a homogeneous medium gives rise to a potential well.

5.2 We consider a homogeneous medium with uniform density $\boldsymbol{\rho}$ and a spherically-symmetric energy scale variation $\boldsymbol{\xi(r)}$. The gravitational acceleration produced by the energy scale variation is given by integrating the acceleration produced by the individual thin shells

$$\ddot{\boldsymbol{r}} = \frac{-\,\boldsymbol{4\,\pi\,G\,\rho}}{\boldsymbol{r^2\,\xi(r)}} \int_{\boldsymbol{0}}^{\boldsymbol{r}} \boldsymbol{x^2\,\{\xi(x)-1\}\;dx} \qquad (32)$$

where the "$\boldsymbol{-1}$" in the integrand accounts for the acceleration produced by the matter alone (ignoring the energy scale variation), which must, of course, be ignored.

5.3 Once the acceleration is known, the energy gained by a particle falling into the potential can be computed by numerically integrating equation (32).

5.4 Figure 4 illustrates the gravitational effects of a Gaussian energy scale variation as defined by equation (2) with $\boldsymbol{\alpha}$=1.0 and $\boldsymbol{\beta}$=5.0. The green curve shows the Gaussian profile. The blue curve is the gravitational acceleration computed from equation (32) The red curve shows the energy released when a particle falls into the gravitation well (computed by integrating equation 32). The red curve has been normalised to be 1.0 at $\boldsymbol{r=0}$ and 6.0 at $\boldsymbol{r=6.0}$ (a change of 5.0).

5.5 The conclusion can be drawn that gravitational potential wells form in a homogeneous medium whenever and wherever energy scale variations are present.

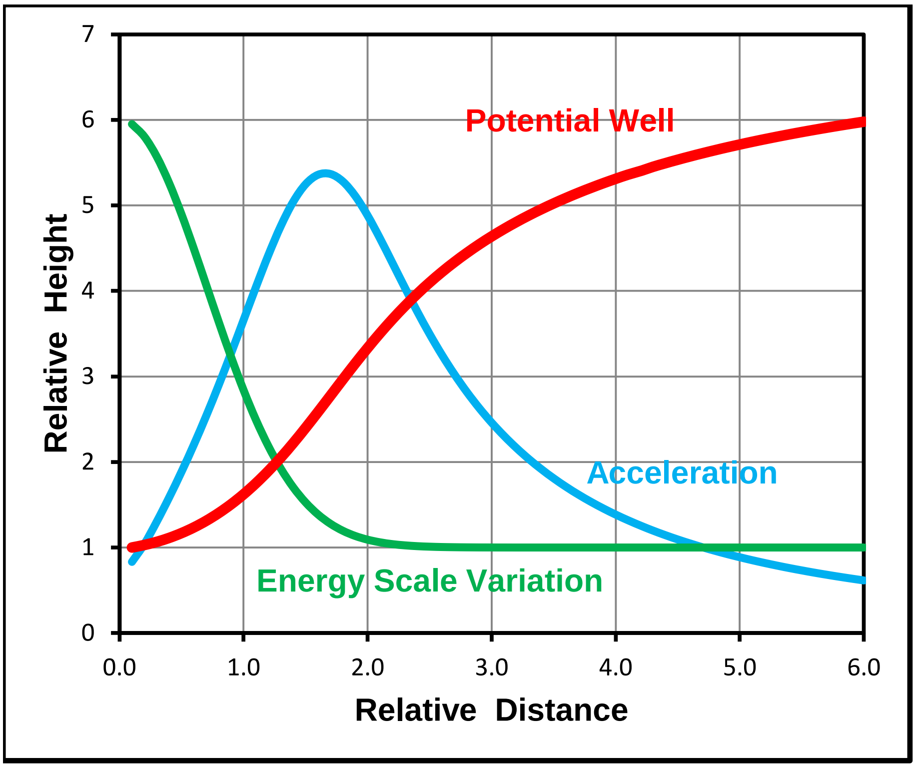

Fig 4. The gravitational potential well arising from a Gaussian energy scale variation in a homogeneous medium. The green curve shows the Gaussian profile of the energy scale variation. The blue curve is the strength of the gravitational acceleration as given by equation (32). The red curve indicates the energy released when a particle falls into the gravitation well (computed by integrating equation 32).

6 Cosmic Microwave Background

6.1 The small scale fluctuations observed in the cosmic microwave background (CMB) are believed to be caused by photons moving in & out of gravitational potential wells formed from clumps of dark matter. Normal baryonic matter could not clump together before recombination because it was fully ionised and any clumps were smoothed out by electron scattering.

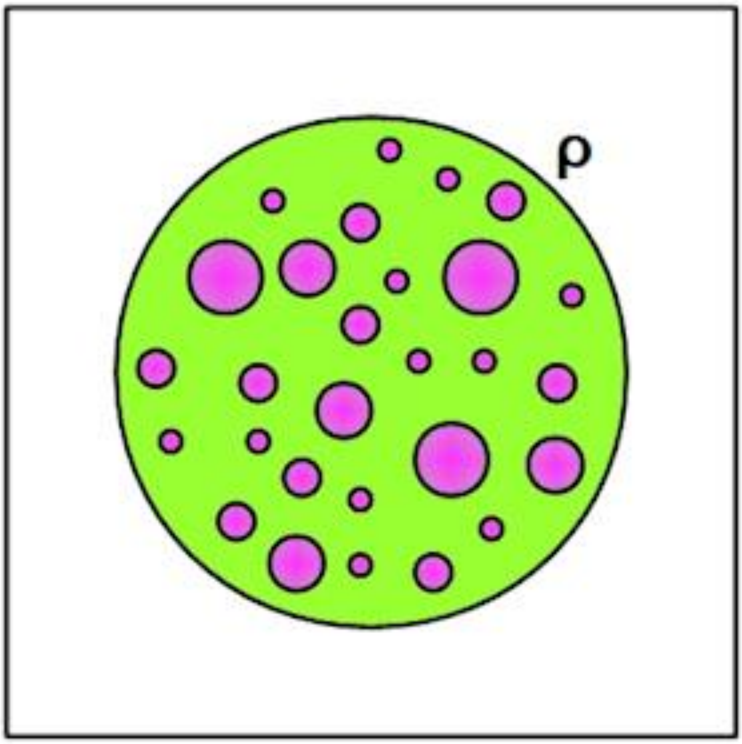

Fig 5. Homogeneous sphere: radius **R**; uniform density **ρ**.; filled with a distribution of energy scale variations (purple disks).

6.2 We now consider a homogeneous sphere, with a uniform density of baryonic matter, and filled with a distribution of energy scale variations of various sizes. This is illustrated in Fig 5.

6.3 Matter outside an energy scale variation experiences a gravitational pull from the matter inside as it behaves as if it has a larger mass. The energy scale variation acts as a gravitational well and gives rise to minute temperature fluctuations in exactly the same way as a gravitational well formed from dark matter.

6.4 Photons entering the energy scale variation gain a minute amount of energy as a gravitational blue shift. On leaving the energy scale variation they lose this energy as a gravitational red shift. This is the Sachs-Wolfe effect (Ryden, 2017) that gives rise to the temperature fluctuations observed in the CMB.

6.5 Baryonic matter pulled into the energy scale variation gains energy and is compressed. Both the temperature and the pressure increase. At some point the pressure rises to such a level that the infall is halted and the material is pushed back out. This gives rise to standing waves usually referred to as acoustic oscillations. These are see in the power spectrum of the cosmic microwave background.

6.6 Here we are suggesting that the baryon acoustic oscillations are caused by the gravitational wells of energy scale variations. We are suggesting that the oscillations are not caused by gravitational wells of dark matter.

7 Discussion

7.1 The derivation of the Friedmann Equation assumes a Universe that is homogeneous and isotropic; i.e. it has the same density everywhere. This is true in our Universe with energy scale variations - the density is the same everywhere. However, in some sense the existence of energy scale variations means the Universe is not homogeneous and isotropic. It would be better to derive the Friedmann Equation afresh and take energy scale variations into account.

7.2 In the usual Friedmann Equation the total energy density is the sum of the energy densities of the individual components (radiation, baryonic matter, dark matter, dark energy). Our hypothesis here leads, not to an addition, but to a multiplication whereby the baryonic matter behaves as if increased by a multiplicative factor.

7.3 The work presented here on the Friedmann Equation gives further support to the cosmology presented in JoKe12 (2017). That paper showed that energy scale variations could account for an expanding Universe that is either decelerating, coasting, or accelerating.

7.4 Current analysis of the cosmic microwave background relies on models that require dark matter to create gravitational wells in the early Universe much earlier than the epoch of recombination. We have argued here that energy scale variations give rise to gravitational wells and that dark matter is not required.

8 References

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