On the variation of the energy scale 12

Cosmology

by

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Summary

The idea has been put forward that the energy scale can vary from location to location. This idea has been used to explain the rotation curves of spiral galaxies and other observations without the need for any dark matter. This paper looks at how variations in the energy scale can be applied to cosmology. The Friedmann equation, the acceleration equation, and the fluid equation can all be recast without invoking either dark matter or dark energy. One result is that energy scale variations allow for a flat matter-only Universe that is accelerating. There is no need for dark matter, dark energy, or a cosmological constant.

1 Introduction

1.1 In Sep 2015 the idea was put forward that the energy scale might vary from location to location (JoKe1, 2015). This idea enabled the rotation curves of spiral galaxies to be explained without the need for any dark matter. JoKe2 (2015) improved the model of JoKe1 to include a Gaussian density distribution for the galaxy and a Gaussian distribution for the energy scale variation. JoKe3 (2015) applied the model of JoKe2 to a set of 74 spiral galaxies. Parameters were obtained that defined the energy scale variations, as well as values for the galaxy masses and rotational velocities.

1.2 Other papers in this series have dealt with: clusters of galaxies; collisions between clusters of galaxies; galaxy interactions; gravitational lensing; primordial density perturbations.

1.3 This paper looks into applying energy scale variations to the basic equations of cosmology. The equations used here are taken from Ryden (2017).

2 The Friedmann Equation

2.1 For a flat Universe (zero curvature) the Friedmann equation is

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\,\pi\,G}{3\,c^2}\varepsilon_c \tag{1}$$

where a is the length scale factor; $\dot{a}$ the rate of change of the scale factor; H the Hubble parameter; ε_c the critical energy density.

2.2 For the Benchmark model of cosmology (Ryden 2017), the critical energy density, ε_c, is given by the sum of the energy densities of the individual components

$$\varepsilon_c = \varepsilon_b + \varepsilon_r + \varepsilon_{DM} + \varepsilon_\Lambda \tag{2}$$

where ε_b is the energy density of baryons (normal matter); ε_r the energy density of radiation; ε_{DM} the energy density of dark matter; ε_Λ the energy density of the cosmological constant, Λ.

2.3 We are assuming no dark matter and no dark energy. So our energy density is simply

$$\varepsilon = \varepsilon_b + \varepsilon_r \tag{3}$$

2.4 To cope with our assumption that the energy scale can vary from location to location we introduce a dimensionless energy scale factor, γ. The Friedmann equation now becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\,\pi\,G}{3\,c^2}\varepsilon\,\gamma \tag{4}$$

The energy scale factor γ is required here because the gravitational field at any given location is dependent on the distribution of matter at other locations.

3 The Equation of State

3.1 In cosmology it is usual to define the equation of state, linking the pressure $\boldsymbol{P}$ with the internal energy $\boldsymbol{\varepsilon}$, by

$$\boldsymbol{P} = \boldsymbol{w}\,\boldsymbol{\varepsilon} \tag{5}$$

where $\boldsymbol{w}$ is a constant for each component type.

3.2 For matter (both normal baryonic matter and dark matter): $\boldsymbol{w = 0}$
For radiation: $\boldsymbol{w = 1/3}$
For the cosmological constant: $\boldsymbol{w = -1}$

3.3 Equation (5) deals with local values; there is nothing that depends on values at other locations. Hence there is no requirement to add the energy scale factor $\boldsymbol{\gamma}$ to this equation; it remains as it is.

4 The Fluid Equation

4.1 The Fluid equation is derived from the first law of thermodynamics with no heat flow.

$$\dot{\boldsymbol{E}} + \boldsymbol{P}\,\dot{\boldsymbol{V}} = \boldsymbol{0} \tag{6}$$

where $\dot{\boldsymbol{E}}$ is the rate of change of internal energy; $\boldsymbol{P}$ is the pressure; $\dot{\boldsymbol{V}}$ the rate of change of volume.

4.2 The internal energy of a volume is the product of the volume with the energy density

$$\boldsymbol{E} = \boldsymbol{V}\,\boldsymbol{\varepsilon_c} \tag{7}$$

4.3 The volume is proportional to the cube of the length scale, $\boldsymbol{a}$. After a little bit of algebra equation (6) becomes

$$\frac{\dot{\boldsymbol{\varepsilon}}_{\boldsymbol{c}}}{\boldsymbol{\varepsilon_c}} + \boldsymbol{3}\,\frac{\dot{\boldsymbol{a}}}{\boldsymbol{a}}\left(\boldsymbol{1} + \frac{\boldsymbol{P}}{\boldsymbol{\varepsilon_c}}\right) = \boldsymbol{0} \tag{8}$$

4.4 Combining equations (8) and (5) we finally obtain:

$$\frac{\dot{\boldsymbol{\varepsilon}}_{\boldsymbol{c}}}{\boldsymbol{\varepsilon_c}} + \boldsymbol{3}\,(\boldsymbol{1} + \boldsymbol{w})\,\frac{\dot{\boldsymbol{a}}}{\boldsymbol{a}} = \boldsymbol{0} \tag{9}$$

This is the Fluid equation.

4.5 The equations that lead up to equation (9) deal with local values; there is nothing that depends on values at other locations. Hence there is no requirement to add the energy scale factor $\boldsymbol{\gamma}$ to this equation; it remains as it is.

5 The Acceleration Equation

5.1 Differentiating equation (1) gives:

$$2\,\frac{\dot{a}}{a}\left\{\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right\} = \frac{8\,\pi\,G}{3\,c^2}\,\dot{\varepsilon}_c \tag{10}$$

5.2 Combining with equations (9) and (1) leads to:

$$\frac{\ddot{a}}{a} = -\frac{4\,\pi\,G}{3\,c^2}\,\varepsilon_c\,(1 + 3\,w) \tag{11}$$

This is the Acceleration equation.

5.3 For matter and radiation the right hand side of equation (11) is always negative. This means the acceleration is always negative and that this Universe is always decelerating. There is no way for this Universe to start accelerating.

5.4 For variations in the energy scale we can do something similar. Differentiating equation (4) gives

$$2\,\frac{\dot{a}}{a}\left\{\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right\} = \frac{8\,\pi\,G}{3\,c^2}\,\varepsilon\,\gamma\left\{\frac{\dot{\varepsilon}}{\varepsilon} + \frac{\dot{\gamma}}{\gamma}\right\} \tag{12}$$

5.5 Combining with equations (9) and (4) leads to

$$\left(\frac{\ddot{a}}{a}\right)\left(\frac{\dot{a}}{a}\right) = -\frac{4\,\pi\,G}{3\,c^2}\,\varepsilon\,\gamma\left\{(1 + 3\,w)\left(\frac{\dot{a}}{a}\right) - \left(\frac{\dot{\gamma}}{\gamma}\right)\right\} \tag{13}$$

5.6 Finally equations (13) and (4) can be combined to give

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{1}{2}\left(\frac{\dot{a}}{a}\right)\left\{(1 + 3\,w)\left(\frac{\dot{a}}{a}\right) - \left(\frac{\dot{\gamma}}{\gamma}\right)\right\} \tag{14}$$

This is our new Acceleration equation.

5.7 The right hand side of equation (14) can become positive by a suitable choice of $(\dot{\gamma}/\gamma)$. Therefore it is possible for this Universe to start accelerating.

6 Equations

6.1 For our conjecture that the energy scale can vary from location to location it is useful to gather the basic equations in one place.

6.2 The Friedmann Equation, equation (4), is

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\,\pi\,G}{3\,c^2}\,\varepsilon\,\gamma \tag{15}$$

6.3 The Equation of State, equation (5), is

$$P = w\,\varepsilon \tag{16}$$

6.4 The Fluid Equation, equation (9), is

$$\frac{\dot{\varepsilon}}{\varepsilon} + 3\,(1+w)\,\frac{\dot{a}}{a} = 0 \tag{17}$$

6.5 The Acceleration Equation, equation (14), is

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{1}{2}\left(\frac{\dot{a}}{a}\right)\left\{(1+3\,w)\left(\frac{\dot{a}}{a}\right) - \left(\frac{\dot{\gamma}}{\gamma}\right)\right\} \tag{18}$$

7 Matter Only Universe

7.1 Models of the Universe show that radiation was dominant over matter for only a short period after the Big Bang. For the majority of the history of the Universe matter has dominated. In our scenario of a flat Universe dominated by matter the equations are as follows.

7.2 The Friedmann Equation, equation (15), is unchanged

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\,\pi\,G}{3\,c^2}\,\varepsilon\,\gamma \tag{19}$$

7.3 The Equation of State, equation (16), is

$$w = 0 \tag{20}$$

7.4 The Fluid Equation, equation (17), is

$$\frac{\dot{\varepsilon}}{\varepsilon} + 3\,\frac{\dot{a}}{a} = 0 \tag{21}$$

7.5 The Acceleration Equation, equation (14), is

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{1}{2}\left(\frac{\dot{a}}{a}\right)\left\{\left(\frac{\dot{a}}{a}\right) - \left(\frac{\dot{\gamma}}{\gamma}\right)\right\} \tag{22}$$

7.6 We look for a solution of equation (22) where both $\boldsymbol{a}$ and $\boldsymbol{\gamma}$ depend on time raised to some power, that is

$$a \propto t^r \tag{23}$$

and

$$\gamma \propto t^s \tag{24}$$

7.7 Feeding these into equation (22) leads to the result

$$r = \frac{s+2}{3} \tag{25}$$

7.8 The expansion of the Universe, equation (23), now becomes

$$\left(\frac{a}{a_0}\right) = \left(\frac{t}{t_0}\right)^{\left(\frac{s+2}{3}\right)} \tag{26}$$

where $a = a_0$ when $t = t_0$.

7.9 Differentiating equation (26) gives the speed of expansion

$$\left(\frac{\dot{a}}{a_0}\right) = \left(\frac{s+2}{3\,t_0}\right)\left(\frac{t}{t_0}\right)^{\left(\frac{s-1}{3}\right)} = H \tag{27}$$

7.10 The Hubble parameter H_0 is defined by $t = t_0$. Equation (27) can be inverted to give the age of the Universe

$$t_0 = \left(\frac{s+2}{3}\right)\frac{1}{H_0} \tag{28}$$

7.11 The behaviour of the energy density follows from equation (21)

$$\left(\frac{\varepsilon}{\varepsilon_0}\right) = \left(\frac{a}{a_0}\right)^{-3} \tag{29}$$

So the energy density depends, as it must, on the inverse cube of the volume. And from equation (26)

$$\left(\frac{\varepsilon}{\varepsilon_0}\right) = \left(\frac{t}{t_0}\right)^{-(s+2)} \tag{30}$$

7.12 Differentiating equation (27) gives the acceleration of the Universe as

$$\left(\frac{\ddot{a}}{a_0}\right) = \left(\frac{s+2}{3\,t_0}\right)\left(\frac{s-1}{3\,t_0}\right)\left(\frac{t}{t_0}\right)^{\left(\frac{s-4}{3}\right)} \tag{31}$$

7.13 In cosmology the deceleration parameter q_0 is defined as

$$q_0 = -\left(\frac{\ddot{a}\;a}{\dot{a}^2}\right)_{t=t_0} \tag{32}$$

Equations (26), (27), (31) lead to

$$q_0 \ = \ - \left(\frac{s-1}{s+2}\right) \tag{33}$$

7.14 We can now consider the behaviour of the Universe using equations (31), (28) and (33) for different values of s.

7.15 $s = 0$

This represents a Universe with no energy scale variations and must give the standard result of a flat matter-only cosmology.

$$\left(\frac{\ddot{a}}{a_0}\right) \ = \ - \left(\frac{2}{9\, t_0^2}\right) \left(\frac{t}{t_0}\right)^{\left(-\frac{4}{3}\right)} < \ 0 \tag{34}$$

This Universe is decelerating and the age of the Universe is

$$t_0 \ = \left(\frac{2}{3}\right) \frac{1}{H_0} \tag{35}$$

and

$$q_0 \ = \ + \ \frac{1}{2} \tag{36}$$

These are the standard cosmological results.

7.16 $s = 1$

The acceleration is

$$\left(\frac{\ddot{a}}{a_0}\right) \ = \ 0 \tag{37}$$

There is no acceleration; the Universe is coasting and expanding at a constant rate. The age of the Universe is

$$t_0 \ = \ \frac{1}{H_0} \tag{38}$$

and

$$q_0 \ = \ 0 \tag{39}$$

7.17 $\boldsymbol{s = 2}$

The acceleration is

$$\left(\frac{\ddot{a}}{a_0}\right) = +\left(\frac{4}{9\,t_0^2}\right)\left(\frac{t}{t_0}\right)^{\left(-\frac{2}{3}\right)} > 0 \tag{40}$$

This Universe is accelerating. The age of the Universe is

$$t_0 = \left(\frac{4}{3}\right)\frac{1}{H_0} \tag{41}$$

and

$$q_0 = -\frac{1}{4} \tag{42}$$

7.18 Figure 1, below, shows the expansion of the flat matter-only Universe for the above three values of $\boldsymbol{s}$. Similar diagrams have been published in many books on cosmology; c.f. Fig 5.2 (Ryden, 2017). The blue (decelerating) and green (coasting) lines are in good agreement. However, the red (accelerating) line is substantially different. In our scenario the acceleration is gentle and not as harsh as the exponential acceleration that comes from a Universe possessing a cosmological constant.

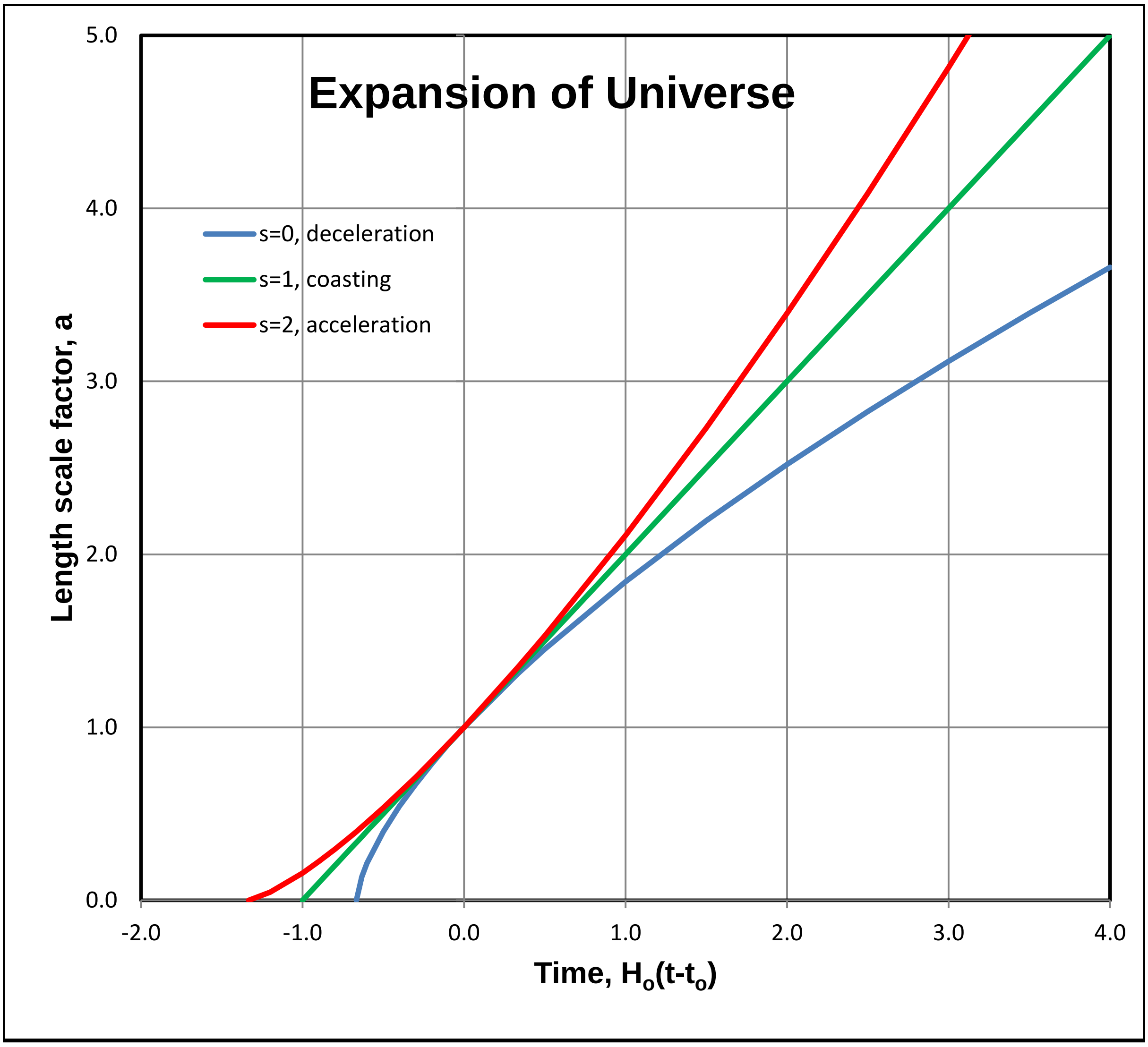

Figure 1: The expansion of a flat matter-only Universe for different values of the energy scale variation parameter, s, as defined in equations (23) & (25). The blue line, s=0, has no energy scale variation and shows the standard deceleration expected for a flat matter-only Universe. The green line, s=1, shows a coasting Universe which expands at a constant rate. The red line, s=2, shows an accelerating Universe. The current epoch (now) is the point: time, $H_o(t-t_o) = 0$; scale factor, a=1.0.

8 Distance and Apparent Brightness

8.1 When we observe distant objects we can measure (amongst other quantities) their red shift and their brightness. In a static or uniformly expanding Universe the Hubble law holds whereby an object's distance is proportional to its red shift. Also the apparent brightness follows the inverse square law. So an object twice as far away has one quarter the brightness.

8.2 In a Universe that is decelerating or accelerating the object's distance is no longer directly proportional to the red shift; nor does the apparent brightness follow an inverse square law. Instead we have to take the deceleration or acceleration into account.

8.3 In a decelerating Universe objects have travelled less than in a uniformly expanding Universe and so they appear brighter than expected.

8.4 In an accelerating Universe objects have travelled further than in a uniformly expanding Universe and so they appear fainter than expected. This is the result derived from observations of distant type Ia supernovae, which are fainter than expected. The usual explanation is that the Universe is accelerating and that 'dark energy' is the cause.

8.5 For small red shifts the luminosity distance $\boldsymbol{D_L}$ of an object is given approximately by (Ryden, 2017)

$$\boldsymbol{D_L = \frac{c}{H_0}\, z \left\{1 - \frac{(1+q_0)}{2}\, z\right\} (1+z)} \tag{43}$$

8.6 Figure 2, below, shows the behaviour of the Universe for the same three cases as in Figure 1. We can compare points in the diagram to the coasting Universe, green curve. For a given red shift points below the green curve have travelled less than their red shift would normally imply. This applies to the blue curve which is our case of a flat matter-only Universe with no energy scale variations (s=0). Points above the green curve have travelled further than expected. This applies to the red line, which is our case of an accelerating Universe (s=2).

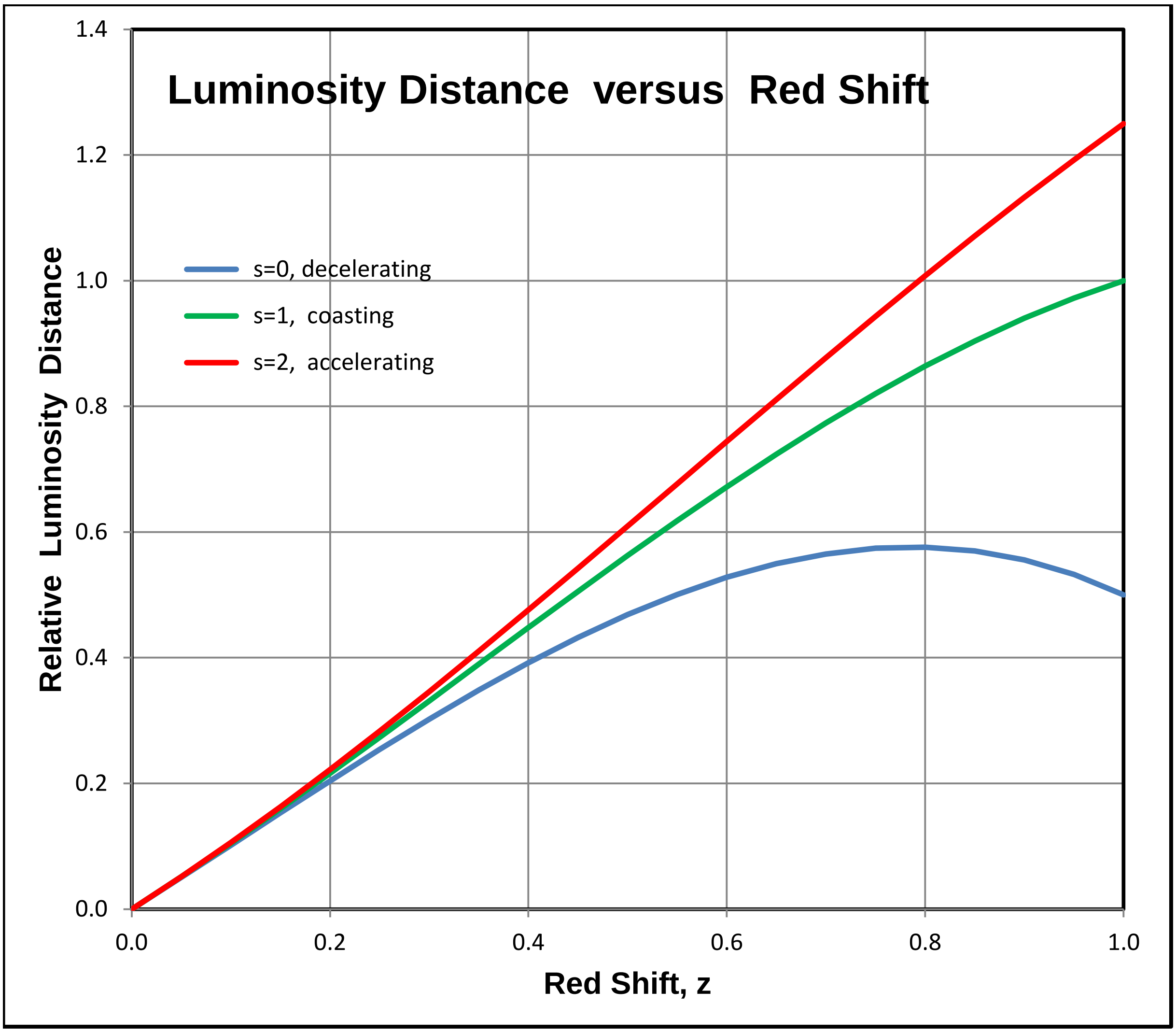

Figure 2. The relative luminosity distance of objects versus their red shift for a flat matter-only Universe. The different curves represent the same cases as in Figure 1. The blue line (s=0) is for no energy scale variations and is a decelerating Universe; at red shift 1.0 it is half the size of the coasting Universe. The green line (s=1) is the coasting Universe and it expands at a constant rate. The red line (s=2) is a Universe that is accelerating; it is 25% larger than the coasting Universe at red shift 1.0.

8.7 The difference in brightness of objects, as defined by the difference in their apparent magnitude $\Delta \boldsymbol{M}$, is given by the log of equation (43)

$$\Delta \boldsymbol{M} \ = \ \boldsymbol{log_{10}}(\boldsymbol{D_L}) \tag{44}$$

8.8 Figure 3, below, shows the magnitude versus red shift behaviour for our three cases. The vertical axis shows the difference in magnitude (i.e. relative values and not absolute values). Larger magnitudes indicate fainter objects. So objects on the red curve are fainter than objects on the green curve, which are fainter than objects on the blue curve. The horizontal axis is the log of the red shift. So point 0.0 is red shift 1.0, and point -1.0 is red shift 0.1.

8.9 For a given red shift points below the green curve (coasting Universe) appear brighter than expected. This applies to the blue curve (s=0), which is our flat matter-only Universe. Objects on this curve at red shift 1.0 would appear 1.5 magnitudes brighter than expected. The observations of remote type Ia supernovae appear to rule out this possibility.

8.10 Points above the green curve appear fainter than expected. This applies to the red curve, which is our flat matter-only Universe with an energy scale variation defined by s=2. Objects on this curve at red shift 1.0 would appear 0.5 magnitudes fainter than expected. The red curve is quite close to the observations of remote type Ia supernovae. It can be compared to similar diagrams shown in most recent books on cosmology (e.g. Figure 6.5, Ryden, 2017).

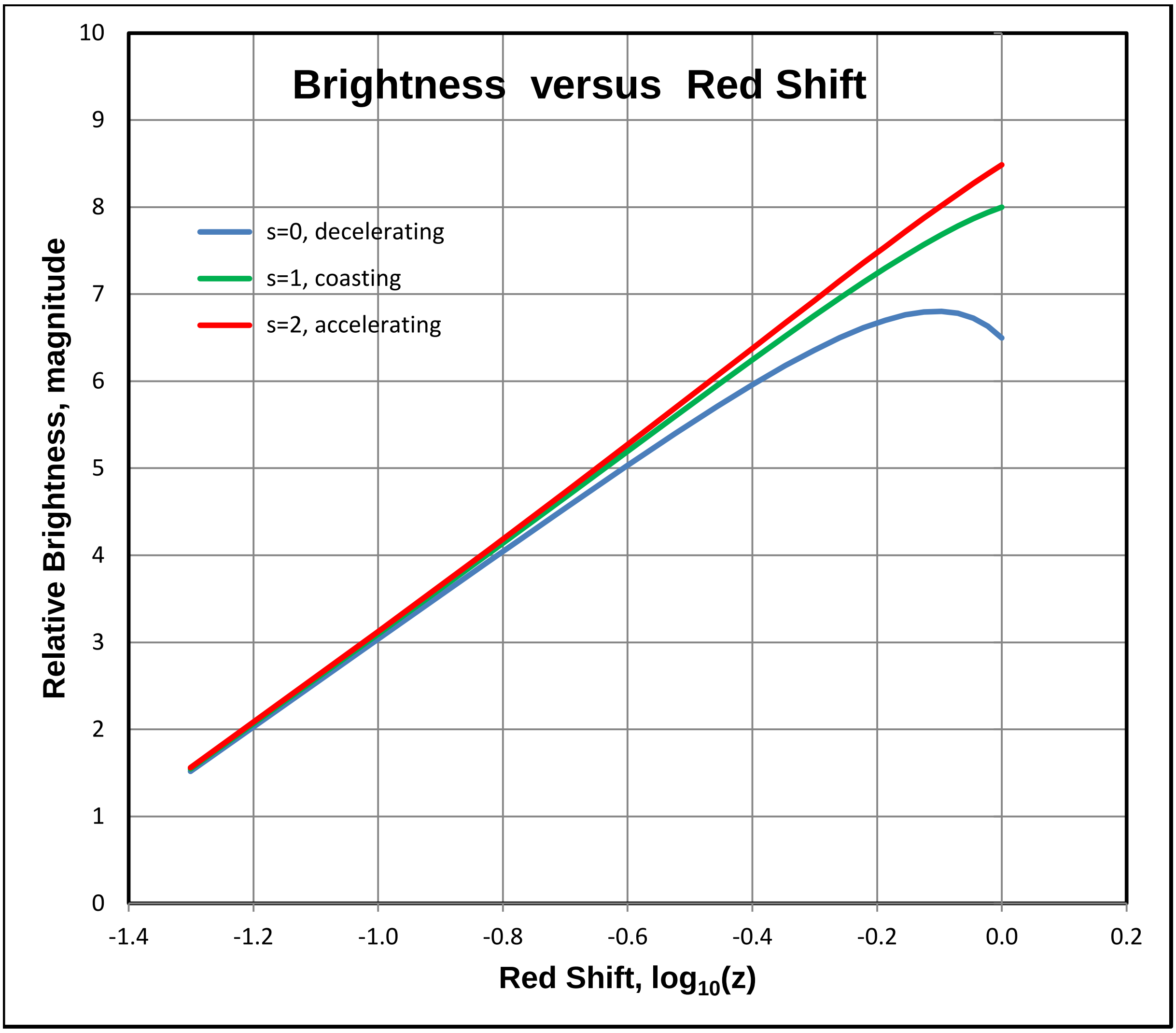

Figure 3. The relation between apparent magnitude (relative brightness) and red shift for a flat matter-only Universe. The different curves are for the same cases as in Figures 1 & 2. The blue line (s=0) is for a decelerating Universe with no energy scale variations. The green line (s=1) is the coasting Universe, with a fixed expansion speed. The red line (s=2) is an accelerating Universe and it matches quite closely the observations of remote type Ia supernovae.

9 Discussion

9.1 The above sections demonstrate that energy scale variations can provide a cosmology without either dark matter or dark energy.

9.2 With the Friedmann equation we have simply replaced the additive terms of dark matter & dark energy with the multiplicative term of energy scale variations. Although this may seem a small technical change it is a huge conceptual change.

9.3 The derivation of the Friedmann equation assumes that the energy density $\boldsymbol{\varepsilon}$ is both isotropic and homogeneous. We have departed from this by introducing the energy scale factor $\boldsymbol{\gamma}$. One way to fix this is to assume that the product $\boldsymbol{\varepsilon\gamma}$ is isotropic and homogeneous or, at least, that deviations are small.

9.4 We introduced the parameter $\boldsymbol{s}$ to define the time variation of the energy scale factor $\boldsymbol{\gamma}$, in equations (23), (24). It is curious that the integer values 0, 1, 2 cover the cases: (a) no energy scale variations; (b) coasting Universe; (c) accelerating Universe that matches the type Ia supernovae. There is no reason, that we are aware of, why $\boldsymbol{s}$ should be an integer.

9.5 The energy scale factor $\boldsymbol{\gamma}$ enables us to explain a flat matter-only Universe where the apparent energy density is only around 5% of the critical density. A value for $\boldsymbol{\gamma}$ of around 20 is sufficient to give a flat matter-only Universe.

10 Conclusion

10.1 The assumption that the energy scale may vary from location to location is sufficient to account for a flat matter-only Universe that is accelerating and where there is no dark matter and no cosmological constant.

10.2 From this paper and previous papers in this series, the single concept of variations in the energy scale is capable of explaining:

a) the energy density of the Universe is only 5% of the critical density
b) the expansion of the Universe is accelerating
c) primordial density fluctuations
d) gravitational lensing by clusters of galaxies
e) collisions between clusters of galaxies
f) the high velocities of galaxies in clusters
g) the flat rotation curves of spiral galaxies

10.3 There is no need for dark matter or dark energy; neither need exist.

11 References

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