On the variation of the energy scale 11

The Gravitational Potential

by

Jo. Ke.

Updated: Sun 11th Feb 2018
Original: Sun 29th Jan 2017
www.varensca.com

Summary

The idea has been put forward that the energy scale can vary from location to location. This idea has been used to explain the rotation curves of spiral galaxies and other observations without the need for any dark matter. This paper looks into possible characteristics of the gravitational field for energy scale variations. For the far field the gravitational potential and gravitational force follow Newtonian gravity but scaled up by a factor $(\mathbf{1}+\boldsymbol{\gamma})$, where $\boldsymbol{\gamma}$ characterises the energy scale variation.

1 Introduction

1.1 In Sep 2015 the idea was put forward that the energy scale might vary from location to location (JoKe1, 2015). This idea enabled the rotation curves of spiral galaxies to be explained without the need for any dark matter. JoKe2 (2015) improved the model of JoKe1 to include a Gaussian density distribution for the galaxy and a Gaussian distribution for the energy scale variation.

1.2 JoKe3 (2015) applied the model of JoKe2 to a set of 74 spiral galaxies. Parameters were obtained that defined the energy scale variations, as well as values for the galaxy mass and rotational velocity.

1.3 Other papers in this series have dealt with: clusters of galaxies; collisions between clusters of galaxies; galaxy interactions; gravitational lensing; primordial density perturbations.

1.4 This paper looks into the form of the gravitational potential.

2 Energy scale variations

2.1 The starting point for many physical theories is a potential, $\boldsymbol{\varphi}$, such that the force, $\boldsymbol{F}$, is given by the gradient of the potential:

$$\boldsymbol{F} = -\boldsymbol{\nabla\varphi} \tag{1}$$

We should try to do something similar with energy scale variations.

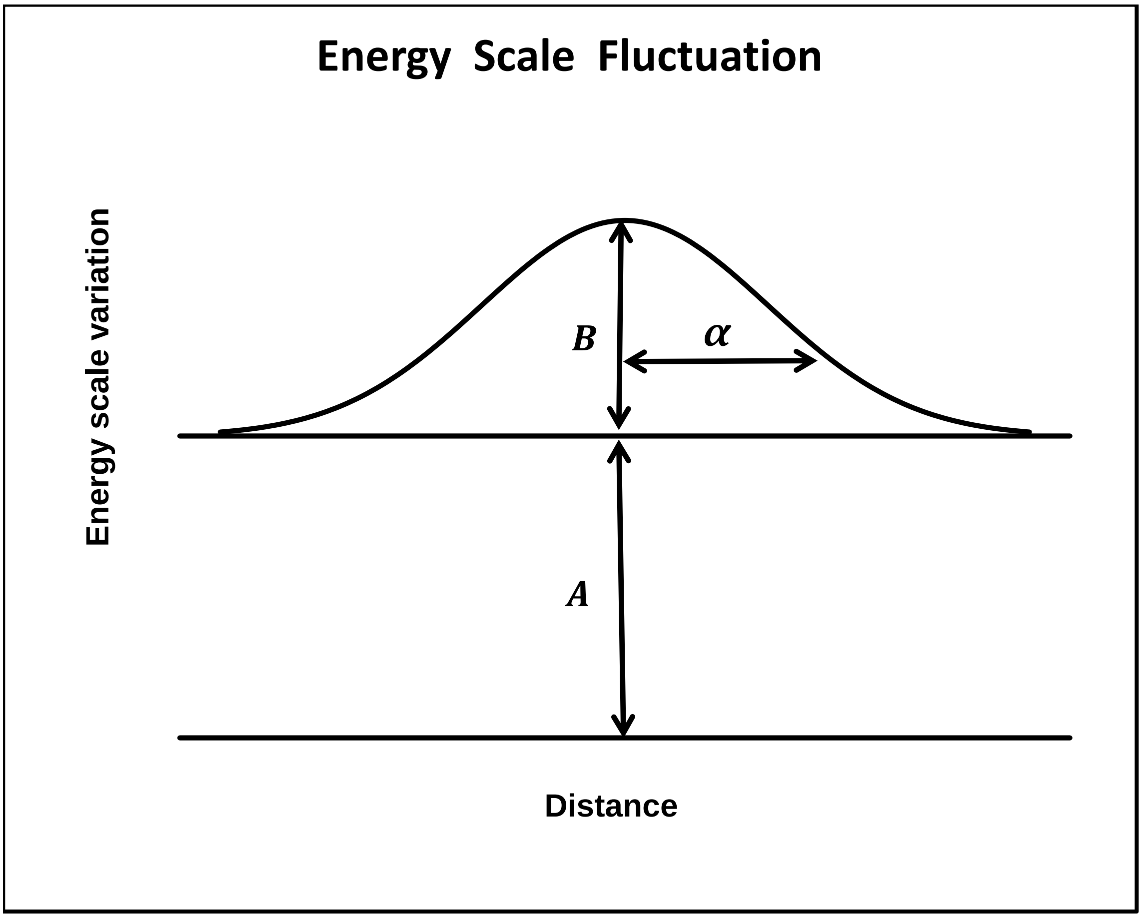

Figure 1. Schematic of an Energy Scale Variation as a Gaussian, height ***B***, 1/e-width $\boldsymbol{\alpha}$, sitting on top of of background with height $\boldsymbol{A}$.

2.2 For energy scale variations we have very little to go on. Previous papers in this series assume a Gaussian profile, where the $\boldsymbol{\xi(x)}$ function is given by

$$\boldsymbol{\xi}(\boldsymbol{x}) = \boldsymbol{A} + \boldsymbol{B}\ \ \boldsymbol{exp}(-\boldsymbol{x}^2/\boldsymbol{\alpha}^2\ ) \tag{2}$$

where $\boldsymbol{A}, \boldsymbol{B}$ are constants for a given energy scale variation; $\boldsymbol{\alpha}$ is the 1/e-width, a distance, see Figure 1.

2.3 It is convenient to write equation (2) as:

$$\boldsymbol{\xi(x) = 1 + \gamma\, exp(-x^2/\alpha^2\,)} \tag{3}$$

where

$$\boldsymbol{\gamma \;=\; 1 + B/A} \tag{4}$$

2.5 The gravitational force, arising from point mass $\boldsymbol{M}$ at distance $\boldsymbol{r}$, is then given by:

$$\boldsymbol{\nabla\varphi = \;-\frac{G\,M}{r^2}\,\frac{(1+\gamma)}{\xi(r)}} \tag{5}$$

2.6 Near the mass, where $\boldsymbol{r \ll \alpha}$,

$$\boldsymbol{\xi(x) \approx 1 + \gamma} \tag{6}$$

and equation (5) simplifies to

$$\boldsymbol{\nabla\varphi \approx \;-\frac{G\,M}{r^2}} \tag{7}$$

This is normal Newtonian gravitation.

2.7 Equation (7) means that there is no need to invoke the existence of dark matter in the centre of galaxies. The normal matter, that is deduced to be there, should be all that is needed to explain the observations.

2.8 Far from the mass, where $\boldsymbol{r \gg \alpha}$, (in practice when $\boldsymbol{r > 3\,\alpha}$)

$$\boldsymbol{\xi(x) = 1 + \gamma\, exp(-r^2/\alpha^2\,)\; \approx 1} \tag{8}$$

and

$$\boldsymbol{\nabla\varphi \approx \;-\frac{G\,M}{r^2}\;(1+\;\gamma)} \tag{9}$$

Again this is normal Newtonian gravitation but scaled up by a factor of $\boldsymbol{(1+\;\gamma)}$.

2.9 Equation (9) can be integrated to give

$$\boldsymbol{\varphi \approx \;-\frac{G\,M}{r}\;(1+\;\gamma)} \tag{10}$$

So the mass behaves as if its mass is a factor $\boldsymbol{(1+\;\gamma)}$ larger than is actually the case.

2.10 In General Relativity the metric for a weak gravitational field is given by:

$$\boldsymbol{ds^2 = -(1 + 2\,\varphi)dt^2 + (1 - 2\,\varphi)(dx^2 + dy^2 + dz^2)} \qquad (11)$$

where

$$\boldsymbol{\varphi = -\frac{G\,M}{r}} \qquad (12)$$

2.11 For energy scale variations we have the same metric but with $\boldsymbol{\varphi}$ given by equation (10). This means all the results of General Relativity for a weak field hold but with the size of the remote mass scaled up by a factor of $\boldsymbol{(1+\gamma)}$.

2.12 It is interesting to note that the potential, $\boldsymbol{\varphi}$, depends only on $\boldsymbol{\gamma}$ and not on the width, $\boldsymbol{\alpha}$, of the energy scale variation. This is a natural consequence of the Gaussian shape, i.e. the height does not depend on the width.

3 The Model

3.1 Rather than work with a point mass we work instead with a thick disk having a Gaussian density distribution. This prevents some quantities from going off towards infinity near the centre. We choose the same model for a disk galaxy as is adopted in papers JoKe2 and JoKe3.

3.2 We assume a disk of thickness h, and a Gaussian density distribution given by:

$$\rho(r) = \rho_o \; exp(-r^2/\beta^2) \tag{13}$$

where ρ_o is the central density; β is 1/e-width of the density distribution.

3.3 The mass, $dM(x)$, of an incremental ring, of width dx and thickness h, is given by:

$$dM(x) = 2\,\pi\,x\,h\,\rho_o\,exp(-x^2/\beta^2)\,dx \tag{14}$$

3.4 The total mass of the galaxy is given by integrating equation (14) from 0 to infinity:

$$M = \pi\,\beta^2\,h\,\rho_o \tag{15}$$

3.5 The mass interior to location r is given by integrating equation (14) from 0 to r:

$$M(r) = M\,\{1 - exp(-r^2/\beta^2)\} \tag{16}$$

4 Gravitational Force

4.1 The gravitational force per unit mass is given by modifying equation (5):

$$\boldsymbol{F(r) \approx -\frac{G\,M}{r^2}\,\frac{(1+\gamma)}{\{1+\gamma\,exp(-r^2/\alpha^2\,)\,\}}\;\{1-exp(-r^2/\beta^2\,)\}} \qquad (17)$$

This ignores the fact that the mass is distributed across regions of different values of $\boldsymbol{\xi}$, but this is a small effect away from the centre.

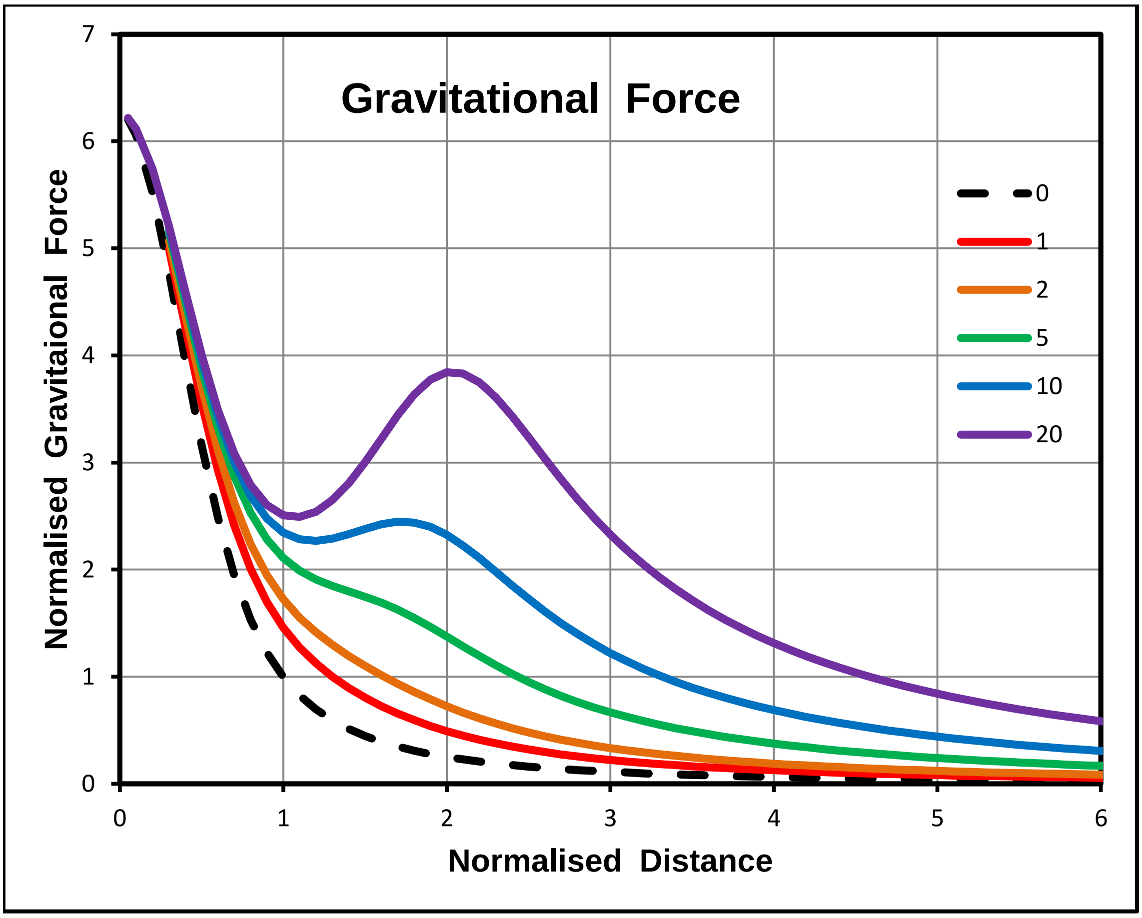

Figure 2. The normalised gravitational force for different values of $\boldsymbol{\gamma}$. The Gaussian density distribution has $\boldsymbol{\beta = 0.4\ \alpha}$, i.e. a narrow density distribution inside a broad energy scale variation.

5 Gravitational Potential

5.1 The gravitational potential is given by integrating (17) from infinity to r:

$$\varphi(r) \approx -G\,M\,(1+\gamma)\int_{\infty}^{r} \frac{\{1-exp(-x^2/\beta^2\,)\}}{\{1+\gamma\,exp(-r^2/\alpha^2\,)\,\}}\,\frac{1}{x^2}\,dx \tag{18}$$

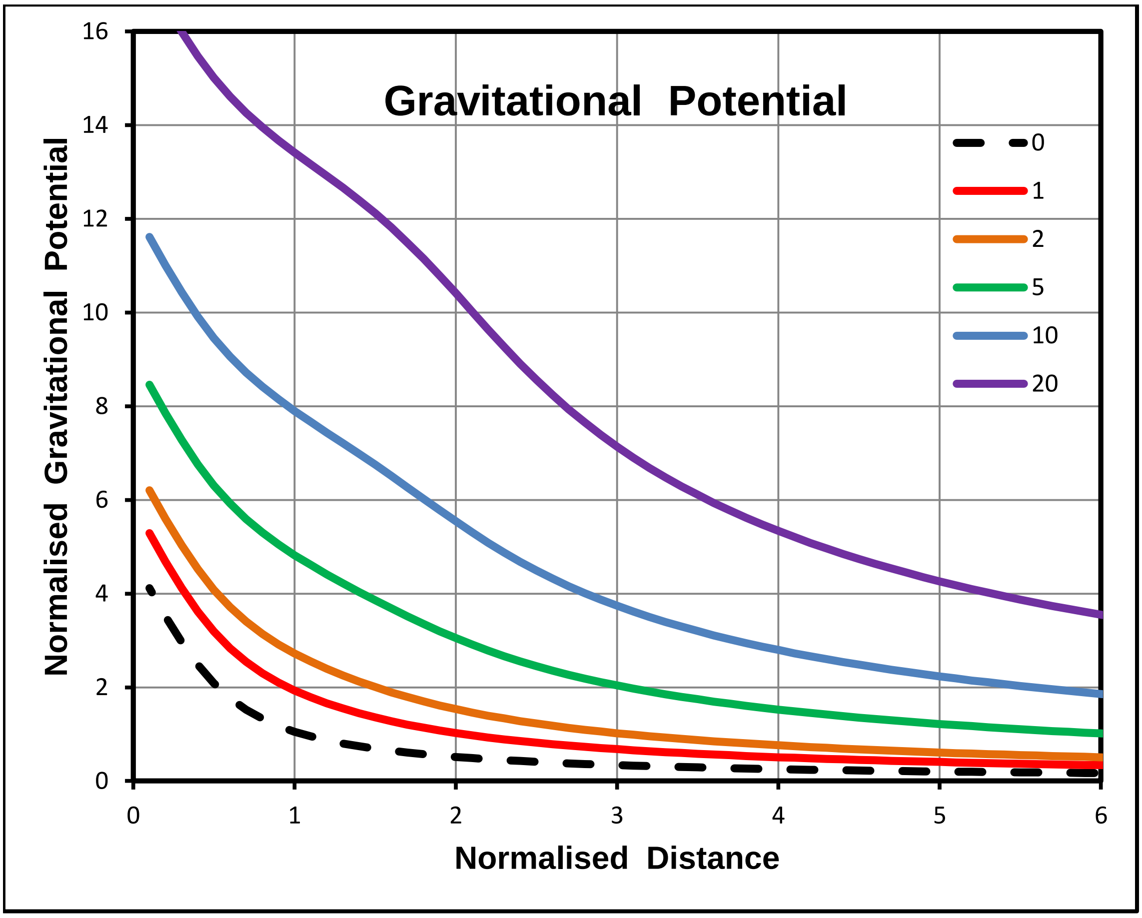

Figure 3. The normalised gravitational potential for different values of γ. The Gaussian density distribution has $\beta = 0.4\ \alpha$, i.e. a narrow density distribution inside a broad energy scale variation.

6 Rotational Velocity

6.1 The rotational velocity is given by equating the centripetal force with the gravitational force. From equation (17) we get:

$$\frac{v(r)^2}{r} \approx \frac{G\,M}{r^2}\,\frac{(1+\gamma)}{\{1+\gamma\,exp(-r^2/\alpha^2\,)\,\}}\{1-exp(-r^2/\beta^2\,)\} \qquad (19)$$

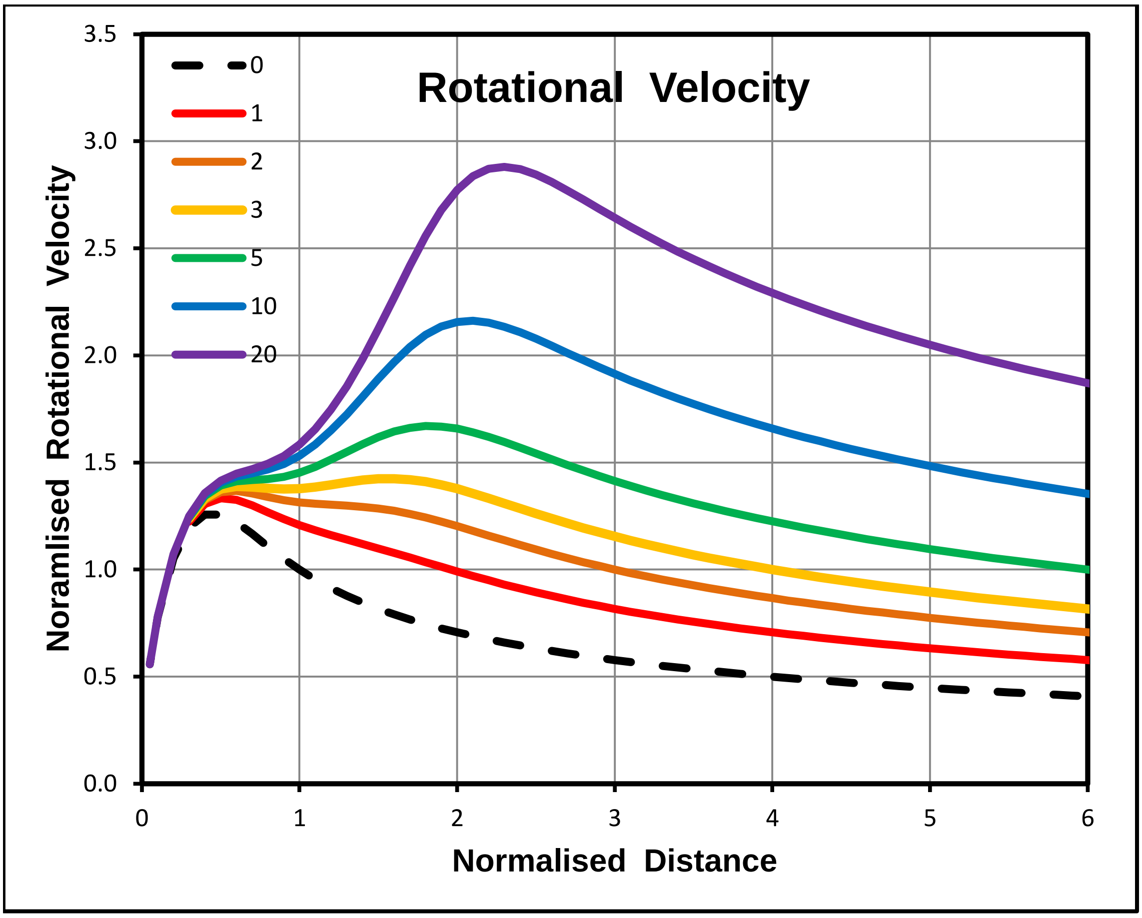

Figure 4. The normalised rotational velocity for different values of γ. The Gaussian density distribution has $\beta = 0.4\ \alpha$, i.e. a narrow density distribution inside a broad energy scale variation.

6.2 It is clear from Figure 4 that, far from the centre, all curves have the expected fall off as $1/\sqrt{r}$.

6.3 The flat rotational curves observed in spiral galaxies appear to be confined to $r < 2\alpha$ and $1 < \gamma < 5$.

7 Angular Momentum

7.1 The angular momentum per unit mass is simply given by

$$r\, v(r) \tag{20}$$

where the rotational velocity $v(r)$ is given by equation (19)

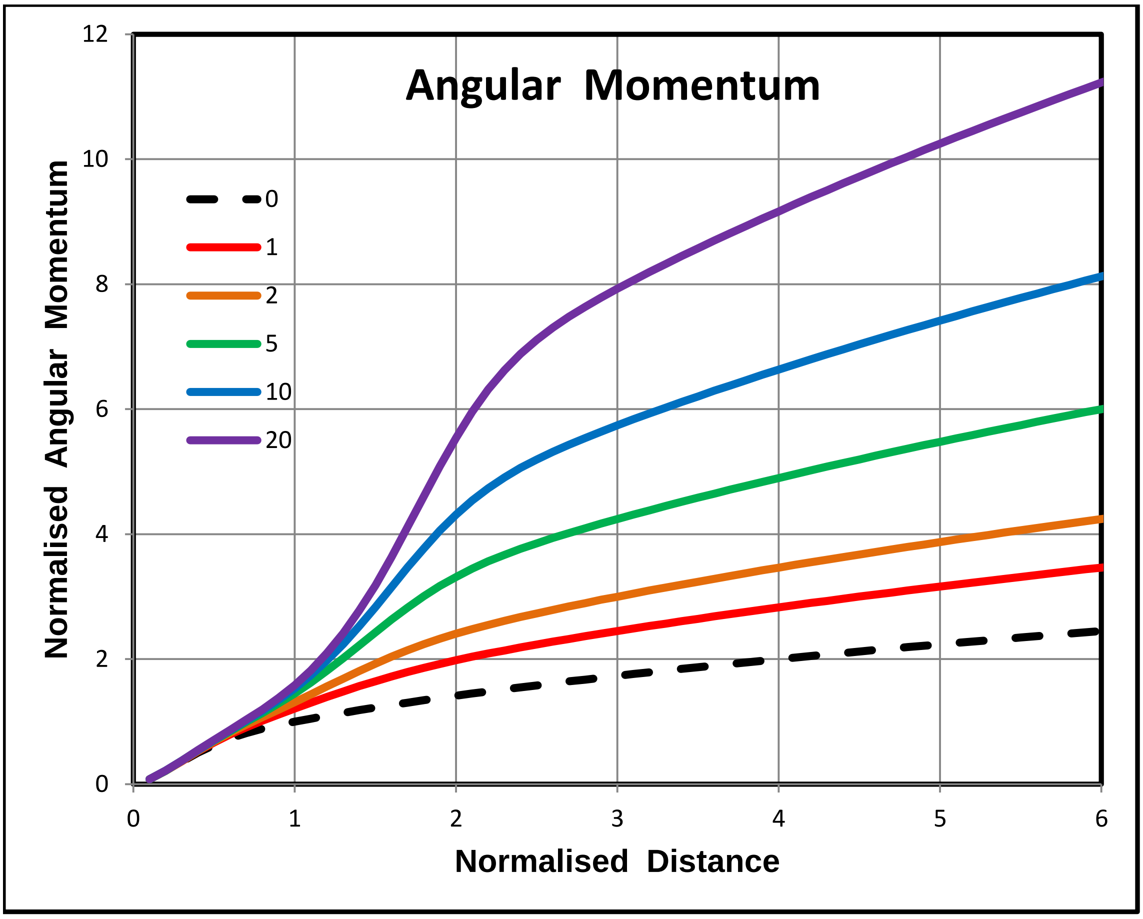

Figure 5. The normalised angular momentum per unit mass for different values of γ. The Gaussian density distribution has $\beta = 0.4\ \alpha$, i.e. a narrow density distribution inside a broad energy scale variation.

7.2 ?

8 Escape Velocity

8.1 The escape velocity is given by the velocity a test particle requires to escape to infinity. The kinetic energy of this velocity comes from the gravitational energy

$$\frac{1}{2}\, m\, v(r)^2 \;=\; m\, \varphi(r) \tag{21}$$

where $\varphi(r)$ is given by equation (18)

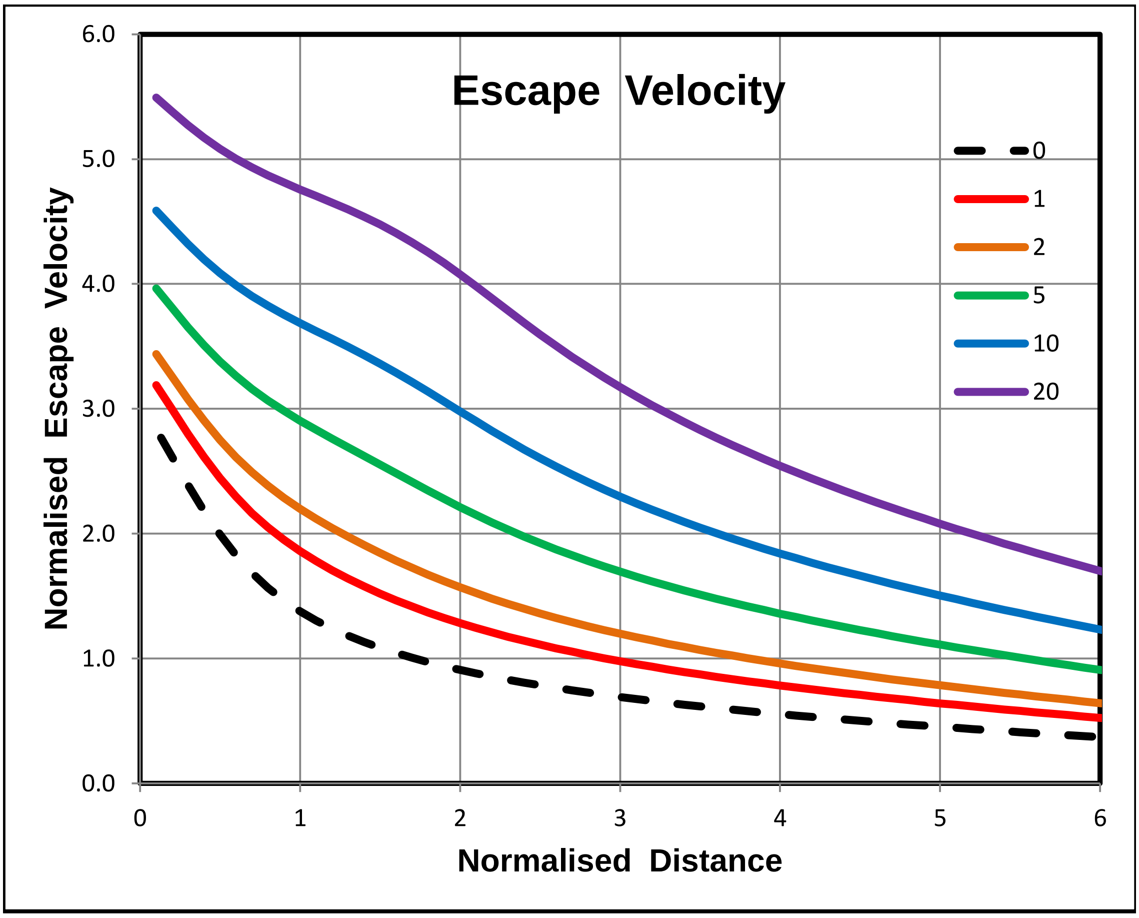

Figure 6. The normalised escape velocity for different values of γ. Plotted is not the true escape velocity, but the velocity required to escape to a distance of 10α. The Gaussian density distribution has $\beta = 0.4\,\alpha$, i.e. a narrow density distribution inside a broad energy scale variation.

8.2 It is clear from Figure 5 that the escape velocity increases with increasing γ. So once an object is captured by a central mass it is much more difficult for it to leave.

9 Conclusion

9.1 Close to the galaxy centre ($\alpha < 0.5$) all the curves are similar to the dashed curve for normal Newtonian gravitation, albeit at a different levels. This reinforces, what has been stated in other papers in this series, namely that variations in the energy scale have little effect in the galactic centre.

9.2 Far from the galaxy centre ($\alpha > 2.5$) all the curves are again similar to the dashed curve for normal Newtonian gravitation, albeit at different levels. In particular the gravitational force reverts to following an inverse square law.

9.3 It is only in the outer regions of a galaxy ($0.5 < \alpha < 2.5$) that variations in the energy scale have any effect that is substantially different from normal Newtonian gravitation. In particular it is only in this region that galaxies can have flat rotation curves.

9.4 So, in effect, it is serendipitous that flat rotation curves are observed in spiral galaxies at all.

10 References

JoKe1. "On the variation of the energy scale: an alternative to dark matter". (Sep 2015). www.varensca.com

JoKe2. "On the variation of the energy scale 2: galaxy rotation curves". (Nov 2015). www.varensca.com

JoKe3. "On the variation of the energy scale 3: parameters for galaxy rotation curves". (Nov 2015). www.varensca.com